DEADBEAT RESPONSE IN SECOND ORDER FEEDBACK CONTROL SYSTEMS

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Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
ELECTRICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California

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ABSTRACT

Many methods have been proposed and tested in the past to obtain an optimal response for step inputs to automatic feedback control systems. Most of these methods have led to the use of sophisticated control devices ranging from small analog to large digital computers. Here the possibility of a simplified switching logic combined with an open-closed loop servomechanism is investigated. Deadbeat response to step inputs was the object of this study rather than a time optimal response. Two types of logic were investigated. A time invariant controller was analyzed, built and tested. The system works on the principle of constant switching times with the output being controlled by an open loop driving voltage which is proportional to the input step size. At the completion of the open loop mode of operation, the system is returned to the normal closed loop mode.

The writers wish to express their appreciation for the assistance and encouragement given them by Dr. John Ward, of the U. S. Naval Postgraduate School, in this investigation.

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1. Introduction.

Numerous schemes have been applied to positioning feedback control mechanisms to obtain optimum response for step inputs. Some of the methods used have been (1) dual mode operation [1] using a relay control computer which approximates the optimal switching line, (2) use of compensation networks to improve relay performance[2], (3) application of discontinuous damping to a relay servo[3], (4) conditional switching techniques[4], and others [5,6]. In general, the basic idea behind these various approaches is the desire to obtain deadbeat response in minimum time for a step input.

The time optimization requirement inherently requires a sophisticated controller that can compute the proper switching times accurately. A question

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[1] K. C. Matthews, R. C. Boe, The application of nonlinear techniques to servomechanisms, National Electronics Conference Vol. VIII, pp. 10-21.

[2] D. McDonald, Nonlinear techniques for improving servo performance, National Electronics Conference, Vol. VI, pp. 400-421.

[3] Harris, McDonald, Thaler, Quasi-optimization of relay servos by use of discontinuous damping, Applications and Industry, November, 1957.

[4] S. I. Leberman, A bang-bang attitude control system for space vehicles, Aerospace Engineering, Oct. 1962

[5] T. R. Frederickson, A time-optimal positioning servo, Control Engineering, February, 1963.

[6] G. J. Thaler, M. P. Pastel, Analysis and design of Nonlinear Feedback Control Systems. Chapter 7.

which might be asked is: "If the requirements for time optimization were dropped, retaining the provision for deadbeat response, would it be possible to design a simple controller for a second order system that would be of practical value?"

In this paper two approaches to the development of a simple controller for a second order system are investigated. In both cases the system operates in two modes, linear and nonlinear, which correspond to closed and open loop modes respectively. In the non-linear or open loop mode a driving voltage is applied to the plant for a prescribed amount of time. The voltage is then reversed until the system reaches an output which either equals or approximates the input. At this point the controller returns the system to the linear, closed loop, operation.

As previously stated, in both approaches investigated, relay switching is caused to occur on a time basis. One of these approaches sets the switching times proportional to the command signal while the other holds the switching times constant while setting the driving voltage. The latter scheme is suggested by the principle of superposition.

The system of Fig. 1 is the basic block diagram for both approaches. The step input commands are represented by R, the system output position by C and the output of the open loop controller by V. The plant is second order

and has the transfer function:

$$(1) \qquad \frac{K/\tau}{s(s+1/\tau)}$$

The system is to be controlled by an open loop controller which functions on command signals only. Any load perturbations or other similar disturbances within the loop will not affect the controller.

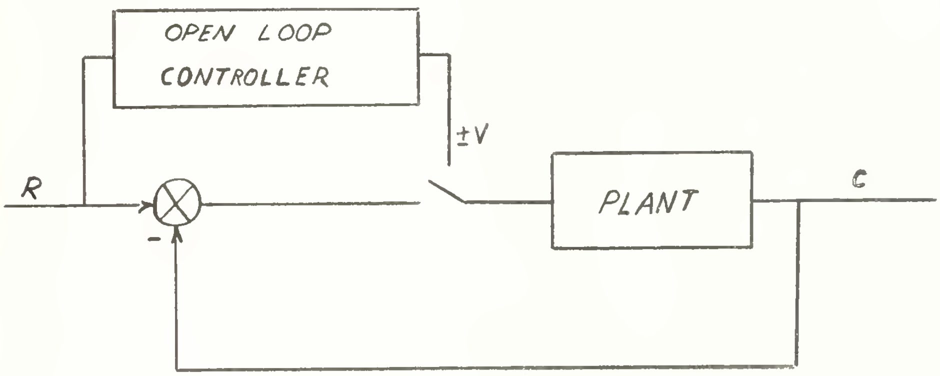


Fig. 1 - Basic System

The assumptions made for the theoretical investigations are (1) the system is linear, (2) the system is second order, (3) command signals are to be steps only and (4) new commands are not to be given until the previous command has been completely executed.

2. Time proportional to command signal.

With the output voltage of an open loop controller a constant in magnitude, there is a specific relationship for the time to reverse the polarity of the driving voltage and the time to return the system to normal linear operation in order to force deadbeat response. By using an approximation to this relationship, a theoretical controller was derived. The system incorporating this controller was then investigated on the CDC digital computer.

For purposes of this investigation, the proposed system is to function as follows: At some time, t_o, a step command signal, $\pm$ R, is applied to the system. Application of the command signal causes the loop to be opened and a voltage, $\pm$ V, to be applied to the plant for a period of time, t_s. At time t_s the polarity of the signal is reversed for a period of time, t_r. The total time of nonlinear operation, t_t, is defined by:

$$(2) \qquad t_t = t_s + t_r$$

At time t_t the controller closes the loop returning the plant to standard closed loop operation. If, at time t_t, the plant output has arrived at the desired position commanded by the input, then true deadbeat response has been achieved.

A digital program was written to compute the times t_s and t_t for various values of inputs. The system parameters, K and τ, as well as the applied voltage, $\pm$V,

were held constant. For the system under evaluation the following values were arbitrarily selected: $K/\tau = 1$, $1/\tau = 5$ and $V = 100$. Each combination of t_s and t_t that results in deadbeat response specifies a command signal R. Thus, from the program data, a plot of t_s and t_t versus R was made for $0 \leq R \leq \pi$ radians as shown in Fig. 2. From these data, t_s and t_t can be approximated by a straight line as shown in the figure. These straight line approximations of t_s and t_t can be evaluated in point slope form as:

$$(3) \qquad t_s = .075|R| + 0.05$$

$$(4) \qquad t_t = .092|R| + 0.105$$

Since t_s and t_t are independent of the sign of the input, the magnitude of R is specified in the above equations.

It is immediately obvious from Fig. 2 that the straight line approximation is extremely poor for small inputs, but this problem can be eliminated if the system is held in closed loop for small inputs. Thus, in this system, the minimum step size for open loop operation would be about .25 radians.

Using the switching times determined from equations (3) and (4), a program was written and appended to "Program

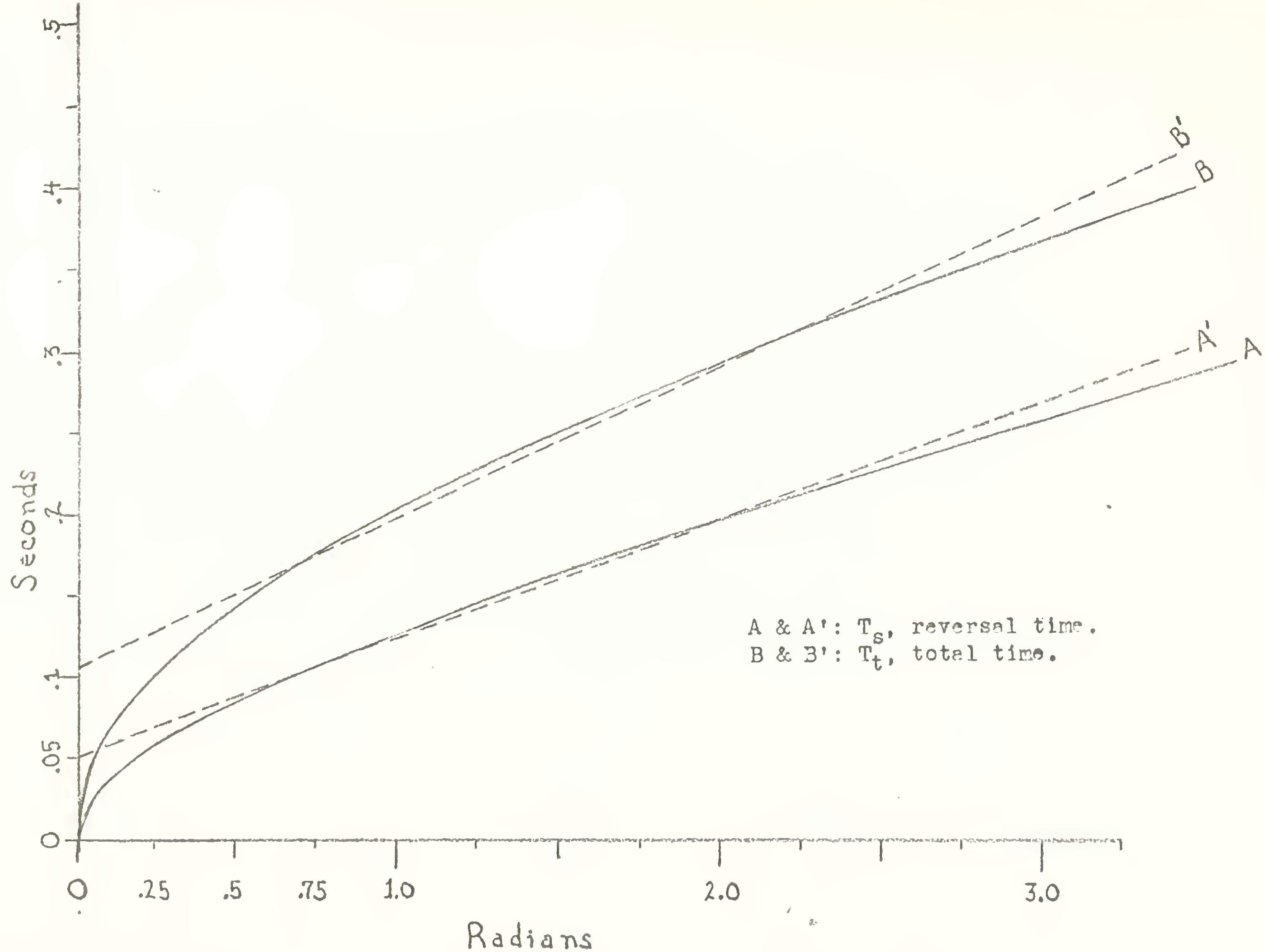


Fig. 2. Command signal versus switching times.

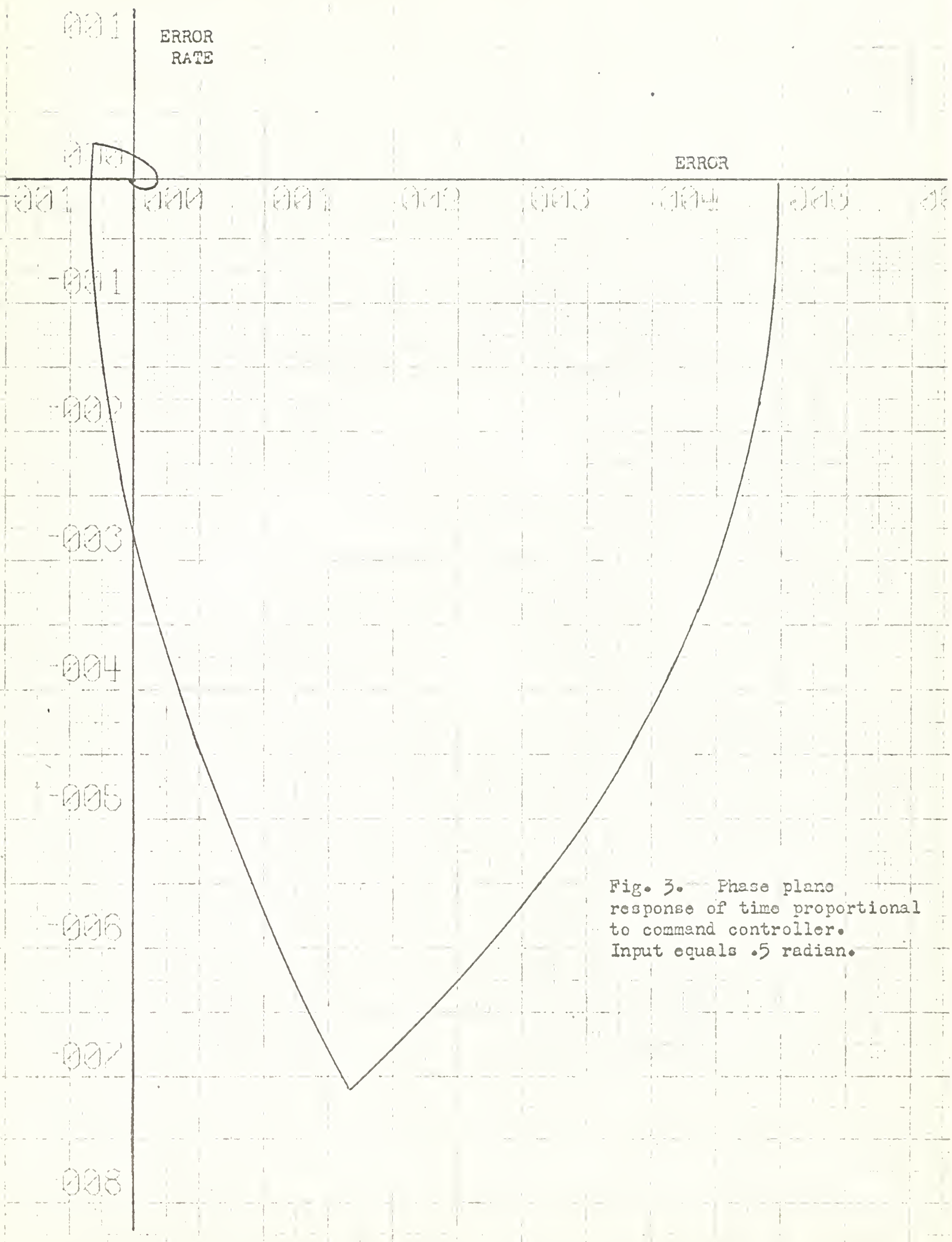


Fig. 3. Phase plane response of time proportional to command controller. Input equals .5 radian.

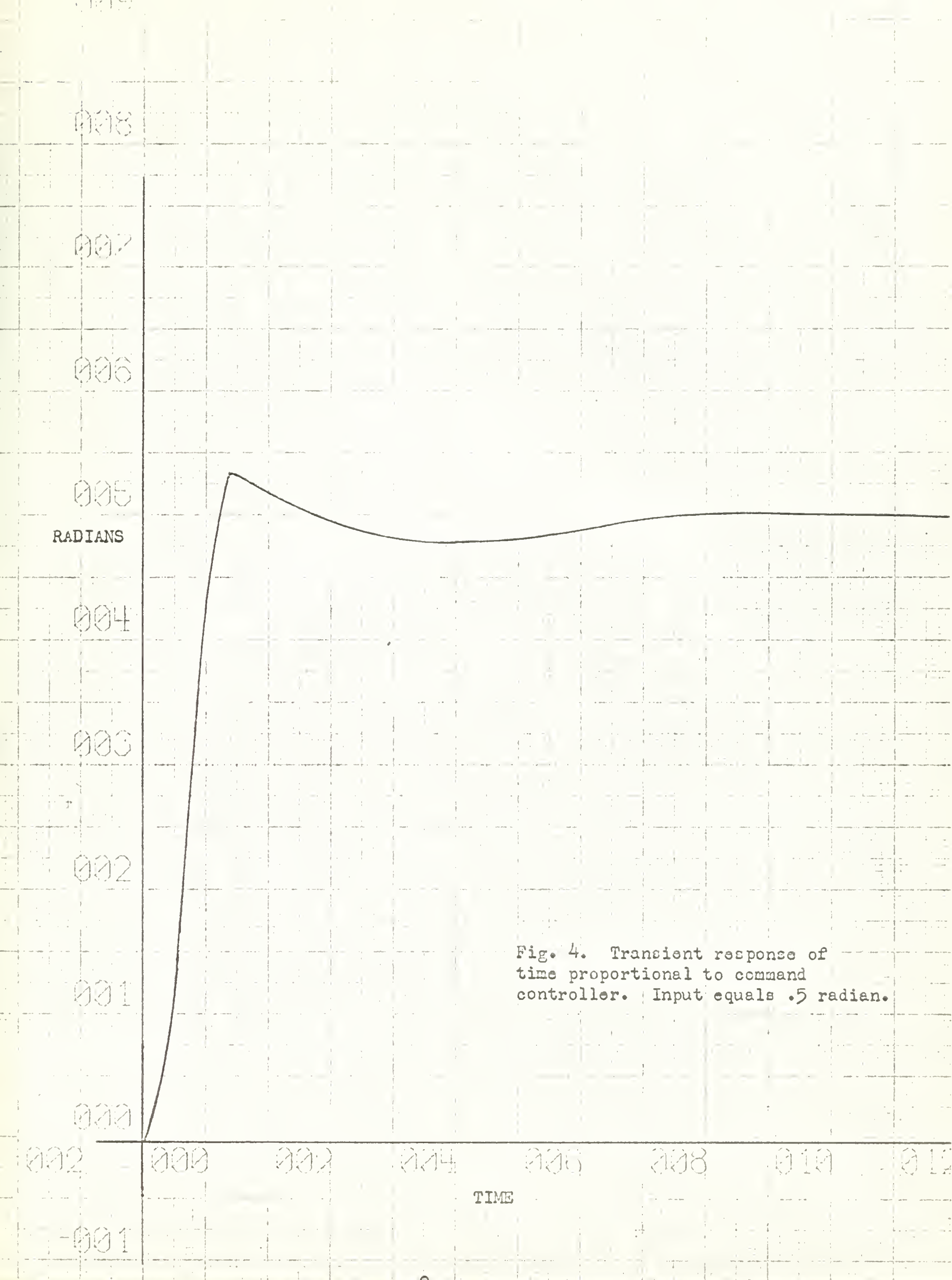


Fig. 4. Transient response of time proportional to command controller. Input equals .5 radian.

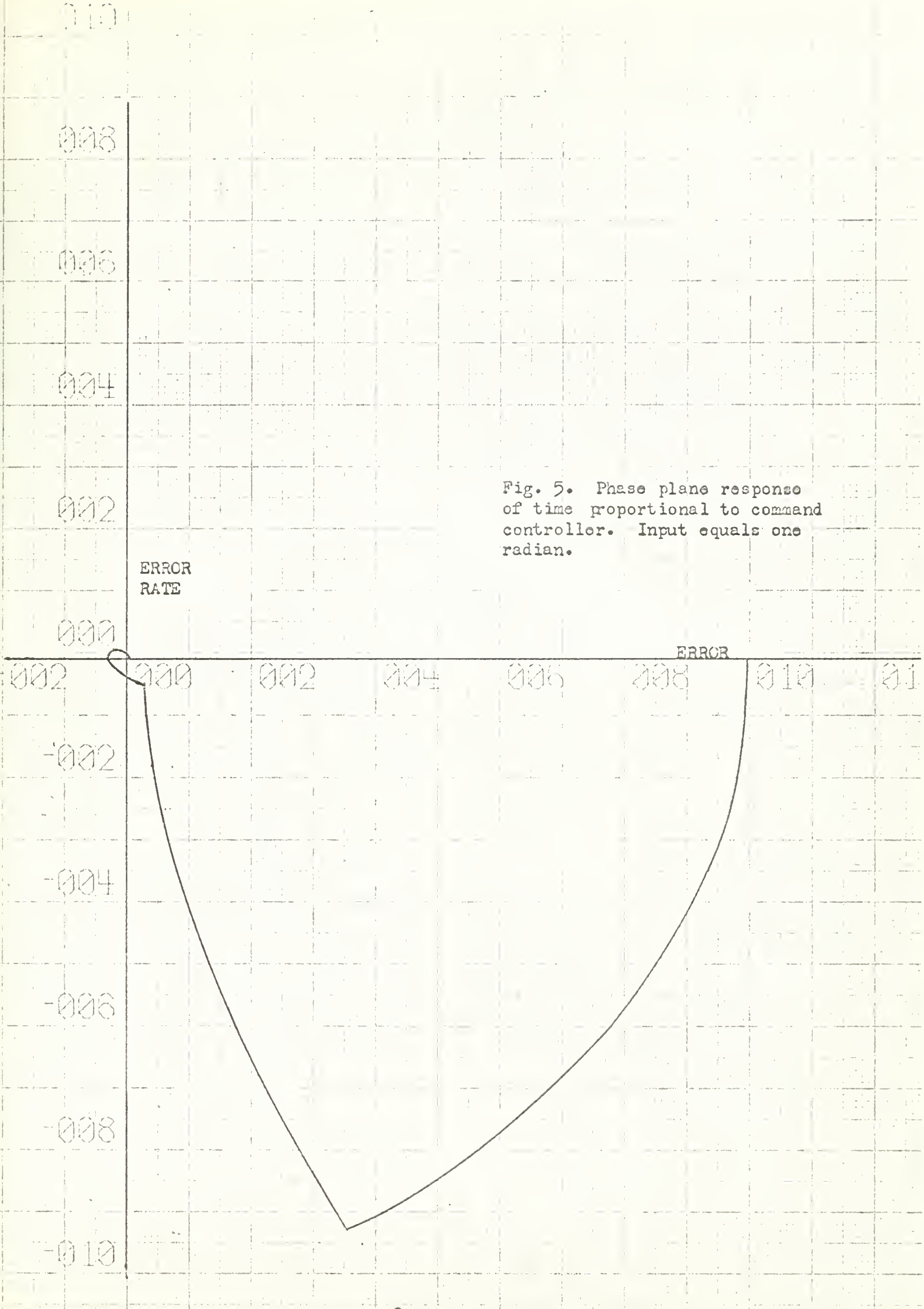


Fig. 5. Phase plane response of time proportional to command controller. Input equals one radian.

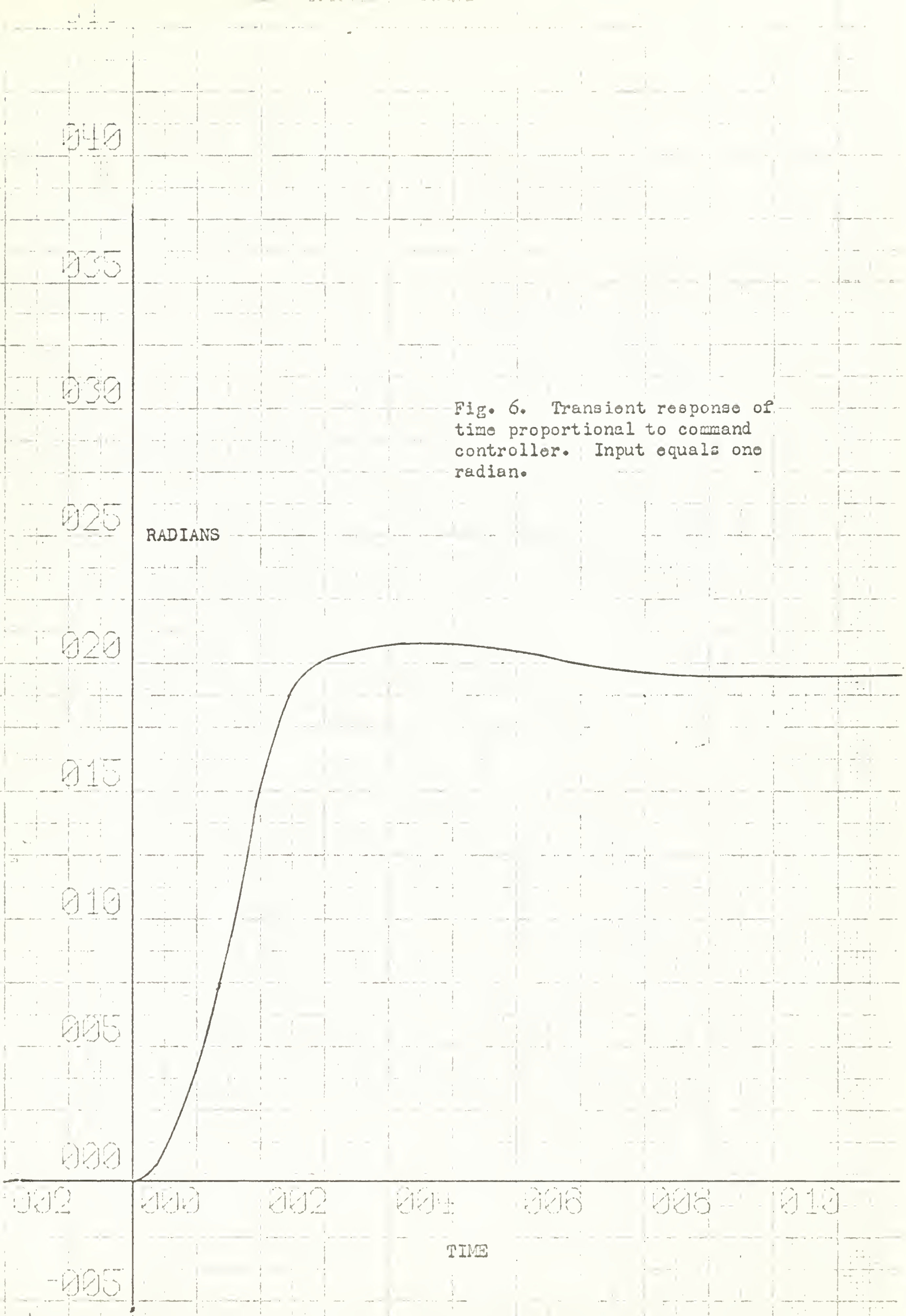


Fig. 6. Transient response of time proportional to command controller. Input equals one radian.

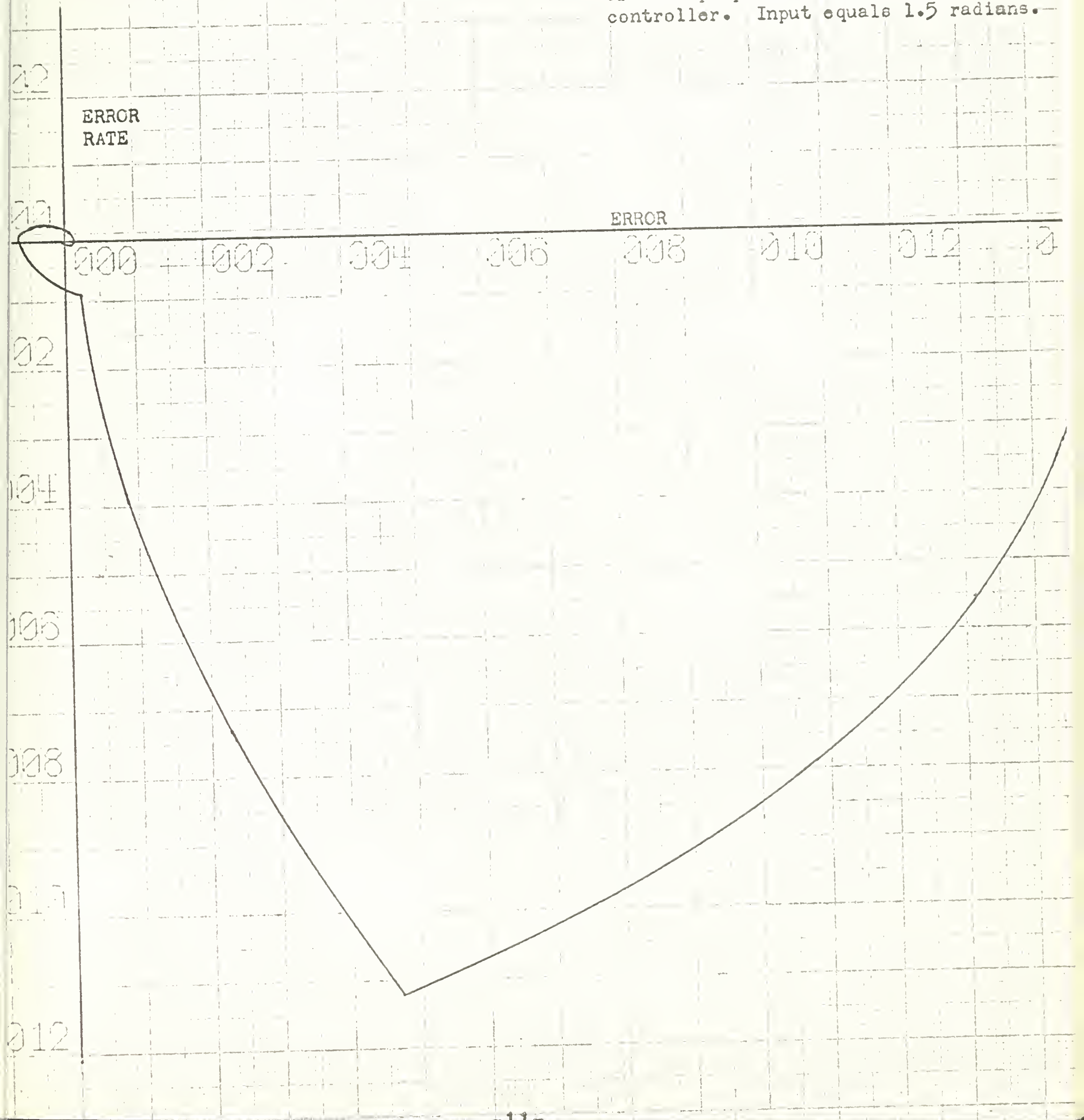


Fig. 7. Phase plane response of time proportional to command controller. Input equals 1.5 radians.

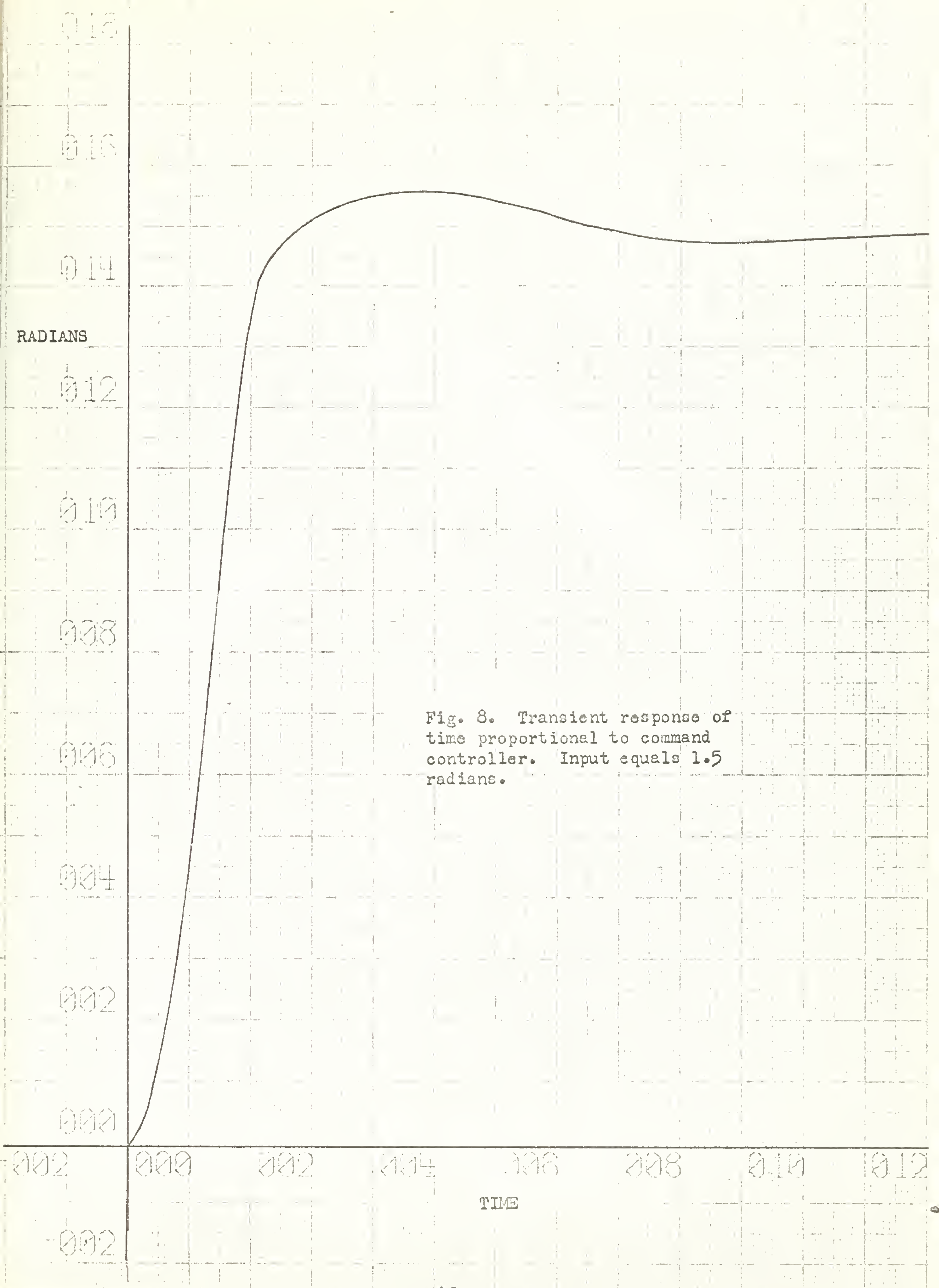


Fig. 8. Transient response of time proportional to command controller. Input equals 1.5 radians.

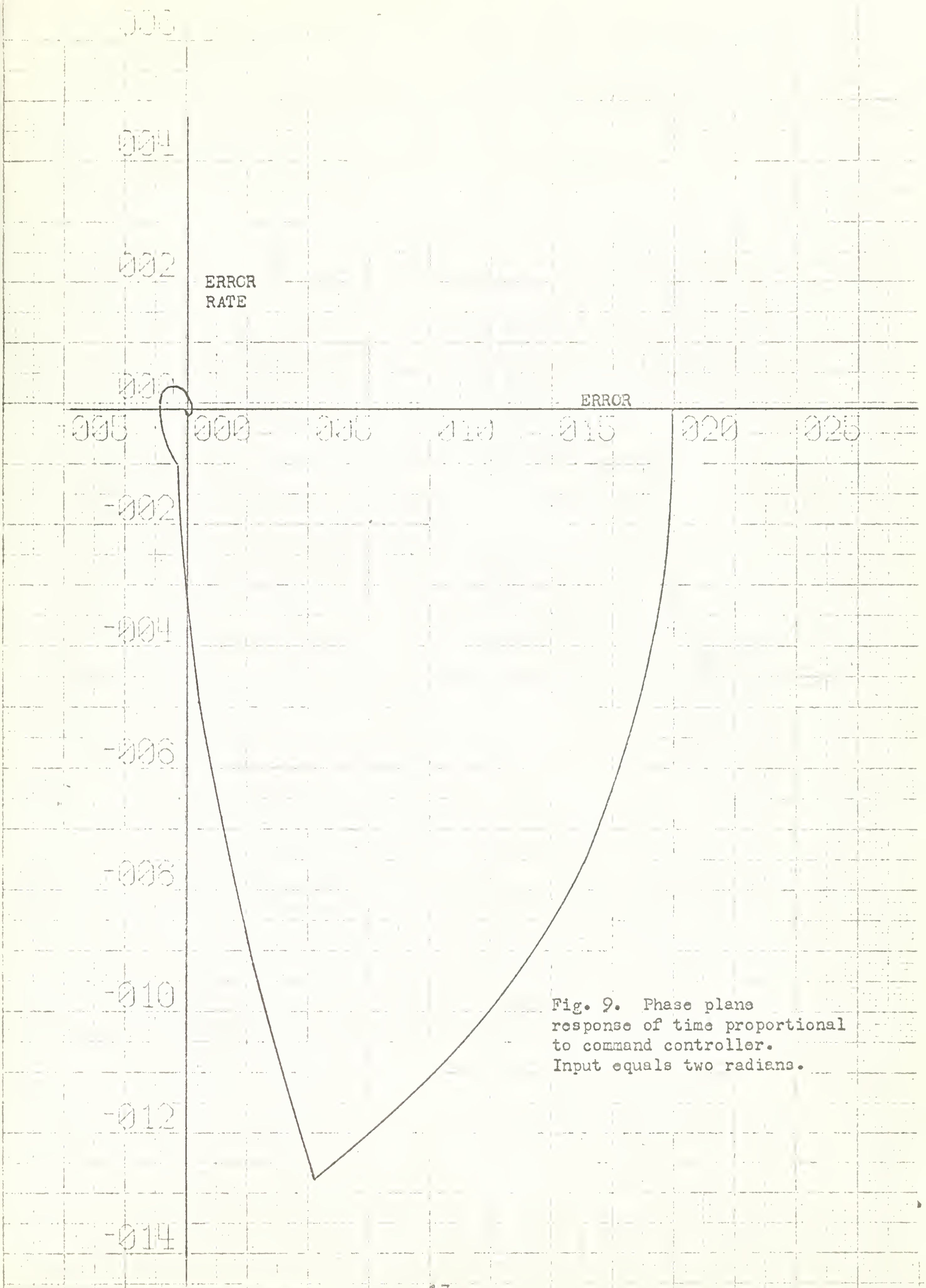


Fig. 9. Phase plane response of time proportional to command controller. Input equals two radians.

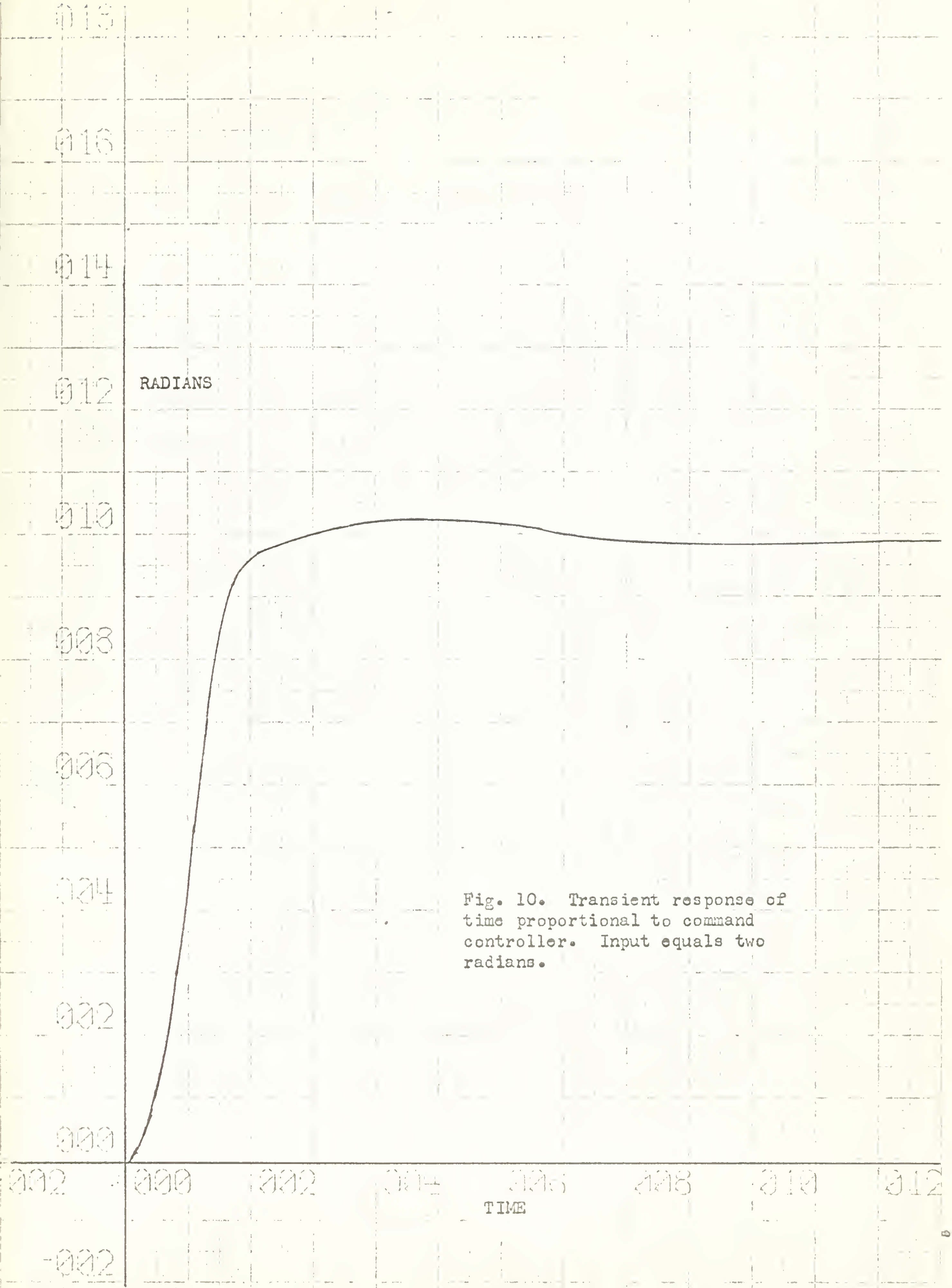


Fig. 10. Transient response of time proportional to command controller. Input equals two radians.

Analog"[7] to study the performance of a system utilizing this type controller. Four runs were made with inputs of .5, 1, 1.5, and 2 radians. The resulting phase planes and time response characteristics as plotted by the computer are shown in Figures 3 to 10. By inspection of Fig. 2 it is possible to predetermine whether switching will occur early or late. For example, if the input is .5 radians, Fig. 2 predicts that both switching times, t_s and t_r, will be late and the system will overshoot. This is verified in Figures 3 and 4. If the input is 1.5 radians, it would be predicted that switching would be early. This is verified in Figures 7 and 8.

This approach does offer some improvement in time-to-steady-state over normal linear operation but it is limited to a comparatively small range of inputs. A controller that could vary the times of switching as specified by equations (3) and (4) would not be simple to construct, which is contrary to one of the main design objectives as set forth in section 1.

* * * * * * * *

[7]"Program Analog" was written by Dr. J. R. Ward of the U. S. Naval Postgraduate School. The program as used is shown in Appendix I.

3. Time invariant controller.

Assume that a given linear second order system is controlled in such a manner as to provide a deadbeat response for a step input. If the output of the linear system is proportional to a step input, then by the principle of superposition[8], the output is doubled when the input is doubled. Furthermore, if the acceleration time, t_s, and the deceleration time t_r, are known for a specified step command, then it will be possible to obtain deadbeat response for all step commands if the switching times are held constant while the driving voltage is varied in proportion to the step command.

To determine the appropriate switching times, it is first noted that if the controller voltage is V, then the system output will, if the initial conditions are zero, be:

$$C(s) = \frac{KV/\tau}{S^2(S + 1/\tau)} \qquad (5)$$

The step voltage +V is to be applied for a finite time, t_s, and the step voltage -V is applied for the finite time, t_r. If at $t_s + t_r$, the output of the system is identically equal to the input, and the output rate is zero, then system response is indeed deadbeat. It is

* * * * * * * *

[8] M. E. Van Valkenburg, Network Analysis, Prentice-Hall, Inc., pp. 79-80.

now necessary to find the required conditions for such deadbeat response.

The inverse transform of equation (5) is:

$$(6) \qquad C(t_s) = KV\left(\tau \epsilon^{-t_s/\tau} + t_s - \tau\right)$$

To obtain the system rate at time t_s it is only necessary to take the derivative of equation (6). Thus,

$$(7) \qquad \dot{C}(t_s) = KV\left(1 - \epsilon^{-t_s/\tau}\right)$$

The reversed voltage is now applied for a period t_r, and considering the initial conditions that exist at time t_s, it follows that the equations for system position and rate for $t_s \leq t \leq t_t$ are:

$$(8) \quad C(s) = -\frac{VK/\tau}{S^2(S + 1/\tau)} + \frac{C(t_s)}{(S + 1/\tau)} + \frac{\dot{C}(t_s)}{S(S + 1/\tau)} + \frac{C(t_s)/\tau}{S(S + 1/\tau)}$$

and:

$$(9) \quad \dot{C}(s) = -\frac{VK/\tau}{S(S + 1/\tau)} + \frac{\dot{C}(t_s)}{(S + 1/\tau)}$$

or:

$$(10) \quad C(t_t) = VK\left(t_s - t_r + \tau\left[1 - \epsilon^{-t_r/\tau}\left(2 - \epsilon^{-t_s/\tau}\right)\right]\right)$$

and:

$$(11) \quad \dot{C}(t_t) = VK\left[-1 + \epsilon^{-t_r/\tau}\left(2 - \epsilon^{-t_s/\tau}\right)\right]$$

For deadbeat response the system rate, $\dot{C}$, must equal zero at time t_t. Therefore, by setting equation (11) to zero, the exact relationship between the

acceleration time, t_s, and deceleration time, t_r, is obtained, namely:

(12) $$\epsilon^{-t_r/\tau} = 2 - \epsilon^{-t_s/\tau}$$

To obtain the deceleration time required for any acceleration time, it is only necessary to take the natural logarithm of both sides of equation (12);

(12a) $$t_r = \tau \ln\left(2 - \epsilon^{-t_s/\tau}\right)$$

To obtain the system output for deadbeat response in terms of t_s and t_r, equation (12) is substituted into equation (10) which yields:

(13) $$C(t_t) = VK(t_s - t_r)$$

Finally, the system output in terms of the accelration time only can be found by substituting equation (12a) into (13) to give:

(14) $$C(t_t) = VK\left[t_s - \tau \ln\left(2 - \epsilon^{-t_s/\tau}\right)\right]$$

Thus by using a controller voltage proportional to R, and by choosing t_s and t_r as calculated from equations (12a) and (14), deadbeat response can be achieved for any step input provided that the plant is adequately described by a (linear) second order transfer function.

4. A design example of the time invariant controller.

A logical sequence of manipulations to determine the proper switching times for a given linear second order system can be easily obtained from equations (12a) and (14). For purposes of illustration, assume that the system has the following characteristics:

$K = 10$	Maximum output desired = 10
$\tau = 1.0$	Saturation will occur if KV is greater than 100.

Using equation (14) with the constants given above and selecting a maximum applied controller voltage of 10 (to avoid the saturation non-linearity) a transcendental equation for the acceleration time, t_s, is obtained:

$$(14) \quad 10 = 100\left[t_s - \ln\left(2 - \epsilon^{-t_s}\right)\right]$$

A value of t_s equal to .368 seconds will satisfy the equation. It is then necessary to find the deceleration time t_r. To do this, substitue time t_s into equation (12a)

$$(12a) \quad t_r = \ln\left(2 - \epsilon^{-.368}\right) = .268 \text{ sec.}$$

The total time of open loop operation is then .636 seconds.

Using the above constants, the system equations were solved on the digital computer. Only one input was given to the system as other inputs would provide the same type of output changed by a constant only. Fig. 11 shows the computed phase plane response of the system.

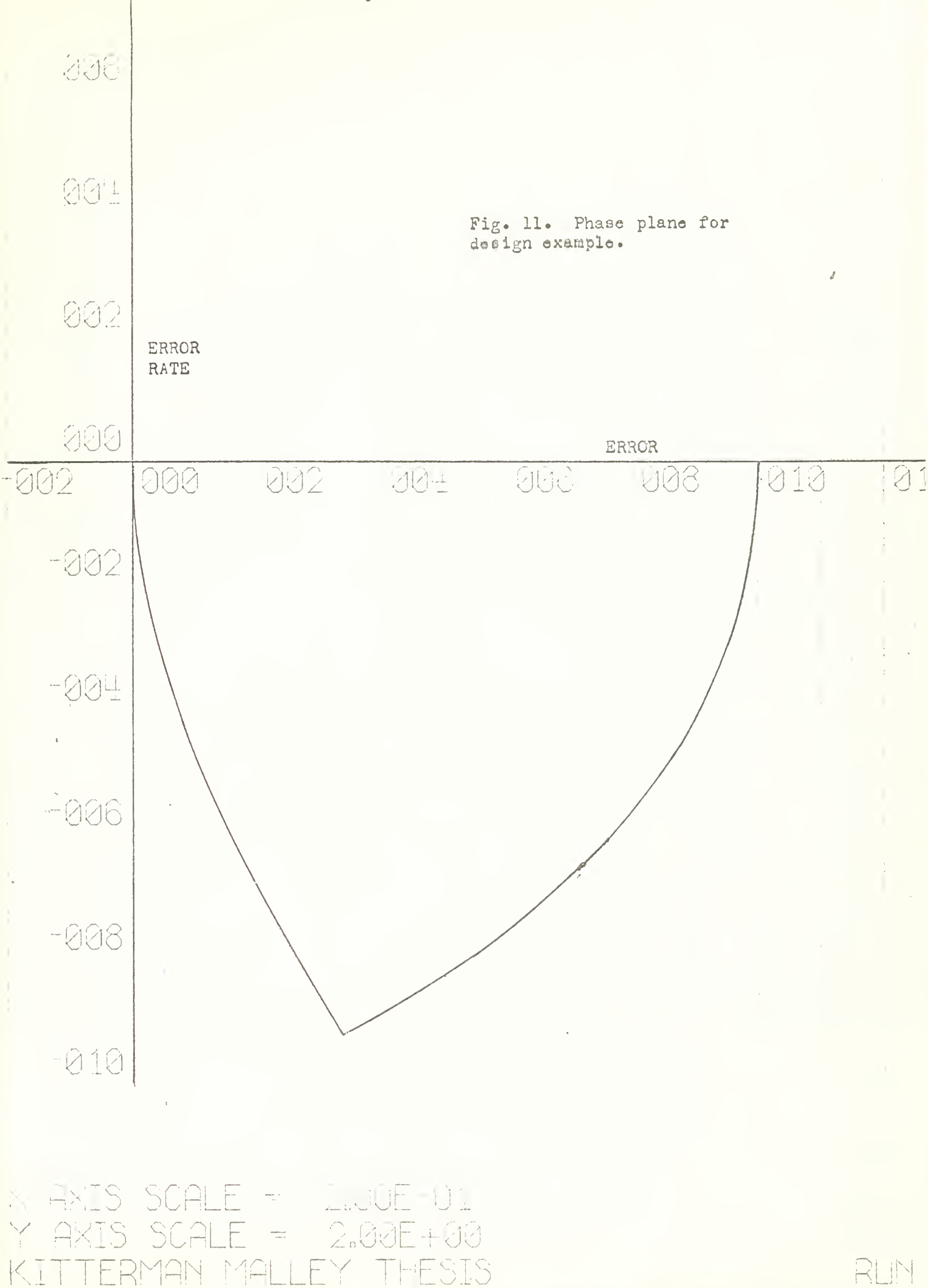


Fig. 11. Phase plane for design example.

5. Test of the time invariant controller applied to a d-c motor plant.

The controller analyzed in sections 3 and 4 above, is here investigated experimentally in relation to a DC motor plant which was predominantly second order and linear up to the saturation limit of the driving amplifier. Runs were made with various step inputs in both the time-invariant and standard closed loop modes.

The following operations by the open loop controller are required to test the theory:

1) At time t_o a step input is commanded. The loop is opened and an open loop driving voltage is applied to the plant.

2) At time t_s the driving voltage is reversed.

3) At time t_t the driving voltage is removed from the plant, the loop is closed, and the system returned to its standard closed loop mode of operation. The operations required were carried out by a timing device consisting of a variable speed D. C. motor geared to a shaft containing three "make and break" switches. The time of contact of these switches relative to each other was variable, that is, their relative anglular positions on the shaft could be varied. The switches were connected to relay circuits which accomplished the desired switching. Thus, relay switching times could be varied by adjusting the speed of the D. C. motor or by adjusting the relative angular position of

the "make and break" contacts.

Fig. 12 shows a block diagram of the experimental D. C. servo used for testing. When a step command is received, relay one switches to its normally open position. This is time t_0. Relay two switches to its normally open position at time t_s to reverse the open loop driving voltage. A third relay, not shown in the figure, is used to return relays one and two to their normally closed positions at time t_t.

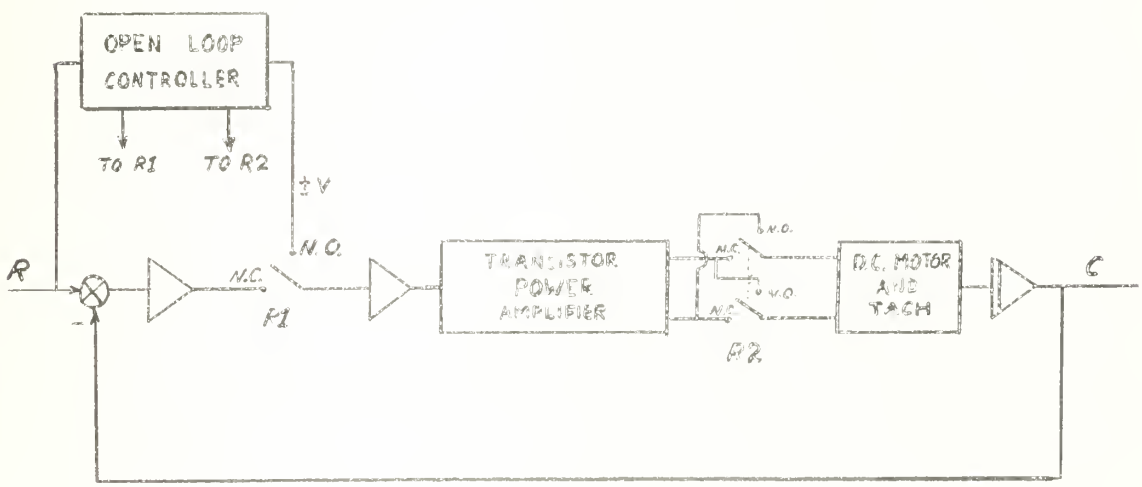


Fig. 12 D. C. Servo with a time invariant controller

The transfer function of the open loop system was evaluated as:

$$(17) \qquad \frac{2.94/1.8}{S(S + 1/1.8)}$$

where the gain is 2.94 and the system time constant is 1.8 seconds. The system output position, C, was obtained by integrating the output of a tachometer geared to the driving motor. As a result, the system output was

evaluated in volts, and consequently, the command and output signals could be compared in volts rather than radians. Further investigation of the open loop system revealed that an open loop command signal of about eight volts would cause saturation in the transistor power amplifier. A schematic of this amplifier is shown in Fig. 13.

For the purpose of steady state accuracy, of primary importance in positioning systems, the closed loop gain was set at 29.4 or ten times greater than the open loop gain.

To simplify the construction of the controller, it was decided to obtain switching times such that a one volt input, R, would cause a one volt driving voltage, V. In other words, the constant of proportionality derived in equation (14) was made equal to one. This means that in equation (14), V = C, and t_s can be evaluated directly from that equation to yield the desired value.

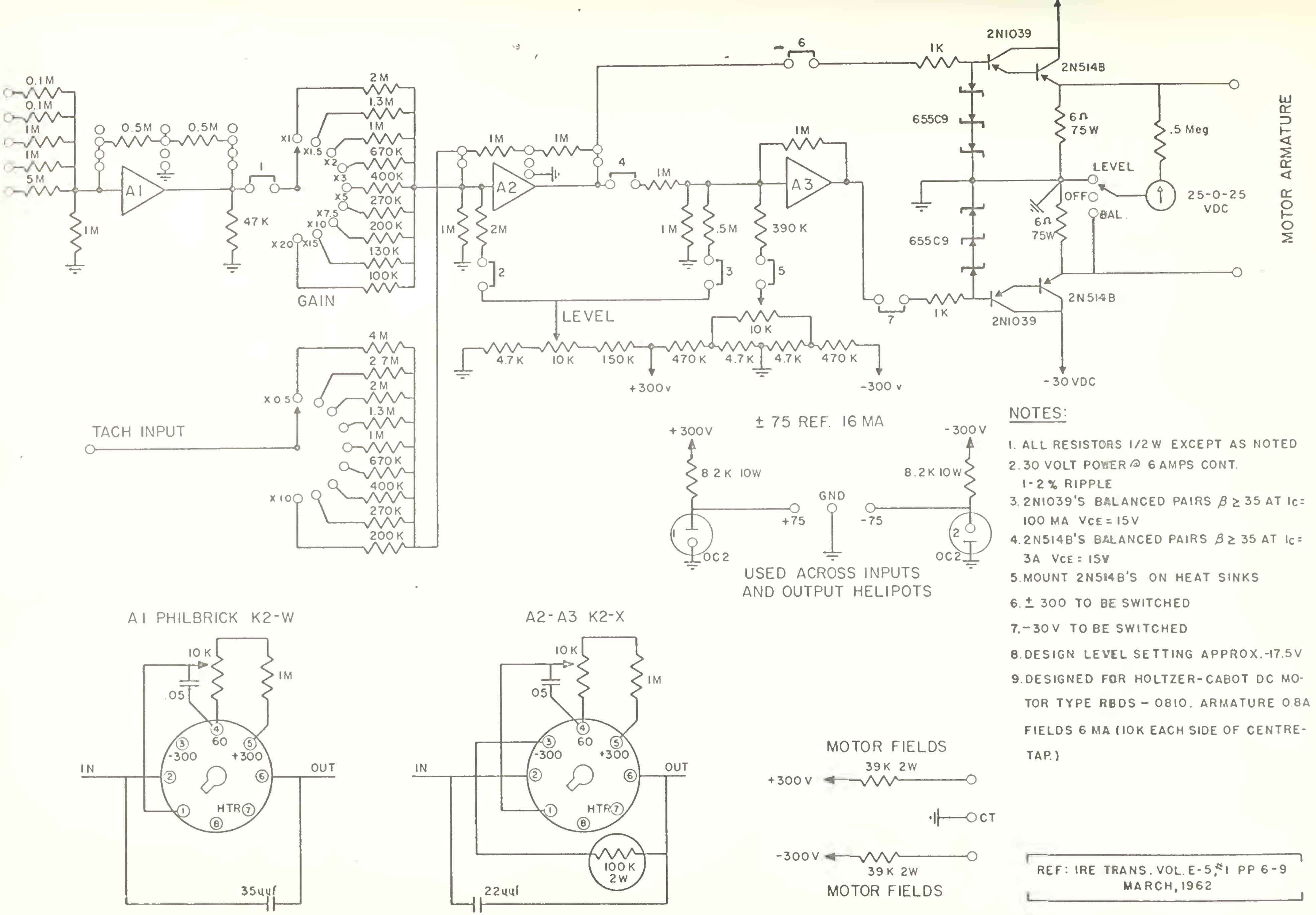
$$(14) \qquad C(t_t) = VK\left[t_s - \tau \ln\left(2 - \epsilon^{-t_s/\tau}\right)\right]$$

$$\frac{1}{2.94} = t_s - 1.8 \ln\left(2 - \epsilon^{-t_s/\tau}\right)$$

$$t_s \cong .98 \text{ sec.}$$

By using the value of t_s equal to .98 sec. in equation (12a), t_r is found to be .54 sec. The relay switching times were then set accordingly.

An input of three volts was used to check out the



NOTES:

1. ALL RESISTORS 1/2 W EXCEPT AS NOTED
2. 30 VOLT POWER @ 6 AMPS CONT. 1-2% RIPPLE
3. 2N1039'S BALANCED PAIRS $\beta \geq 35$ AT I_C = 100 MA V_{CE} = 15V
4. 2N514B'S BALANCED PAIRS $\beta \geq 35$ AT I_C = 3A V_{CE} = 15V
5. MOUNT 2N514B'S ON HEAT SINKS
6. ± 300 TO BE SWITCHED
7. -30V TO BE SWITCHED
8. DESIGN LEVEL SETTING APPROX. -17.5V
9. DESIGNED FOR HOLTZER-CABOT DC MOTOR TYPE RBDS - 0810. ARMATURE 0.8A FIELDS 6 MA (10K EACH SIDE OF CENTRE-TAP.)

REF: IRE TRANS. VOL. E-5, #1 PP 6-9 MARCH, 1962

Fig. 13. Schematic of transistor amplifier.

switching time computed above. The response was not deadbeat. The switching times were then adjusted to obtain deadbeat response to the three volt input. The actual time of voltage reversal was 1.15 sec. compared to the computed .98 sec. The total time of operation was 1.65 sec. compared to the computed time of 1.62 sec. These differences were probably due primarily to the fact that the system was only quasi linear and second order, and the system gain and time constant were determined assuming a linear second order system.

A six channel Brush recorder was used to record the closed loop command signal, the open loop command signal, the system output and the system rate. Runs were made with the inputs varying from 1.4 volts to 10 volts. A standard closed loop response to the same step commands was also made for comparison purposes. Figures 14 through 21 show the responses obtained. Each page contains a time-invariant system response followed immediately by a standard closed loop response to the same signal. Table I is a comparison of the different runs.

TABLE I

D.C SERVO	OPEN-CLOSED LOOP		CLOSED LOOP		
Input	M_{pt} [1]	t_{ss} [2]	M_{pt}	t_{ss}	Δt_{ss} [3]
1.4 volts	1.33	3.8 sec.	1.57	4.8 sec.	1.0 sec.
1.95	1.00	1.8	1.94	6.2	4.4
3.05	1.00	1.7	1.80	6.8	5.1
4.00	1.15	4.6	1.83	7.8	3.2
5.00	1.16	4.5	1.72	7.5	3.0
6.30	1.19	5.6	1.74	8.9	3.3
8.40	1.08	4.8	1.63	9.7	4.9
10.00	1.10	4.7	1.58	10.1	5.4

Note:

1. M_{pt} Ratio of maximum output to desired output.
2. Time to steady state operation, taken to be when system is within .1 volts of command signal.
3. Δt_{ss} is defined as the difference between the times to steady state for the open-closed loop and closed loop operation.

Deadbeat response, or near deadbeat response, could be forced using the time invariant controller if the command signals were in the range of 1.95 to 3.5 input volts. Figures 14 and 15 represent the typical responses obtained in this range. For these recordings the upper trace is the closed loop command signal. Trace number two represents the open loop command signal.

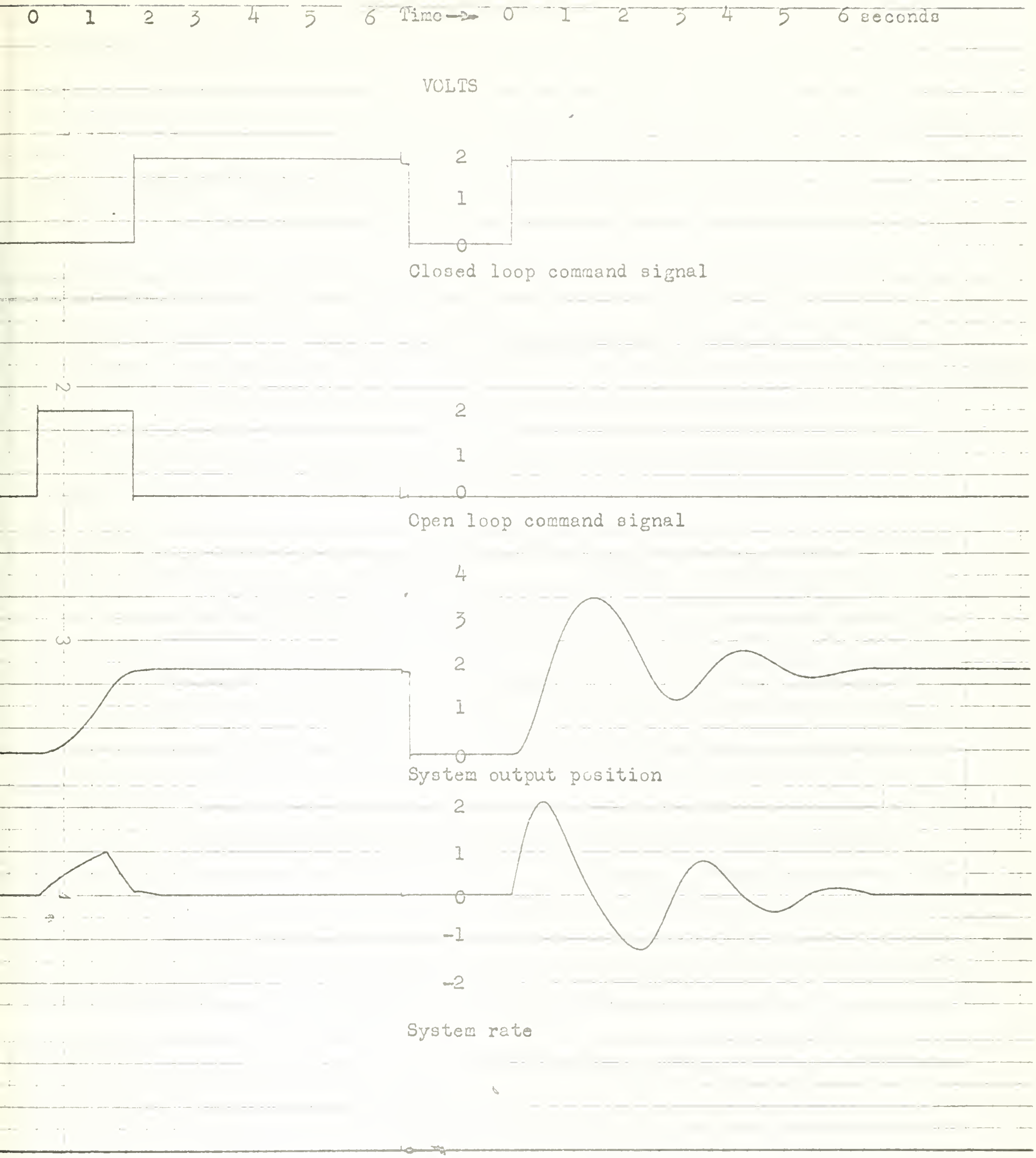


Fig. 14. Time invariant and closed loop response of DC servo system with a 1.94 volt input.

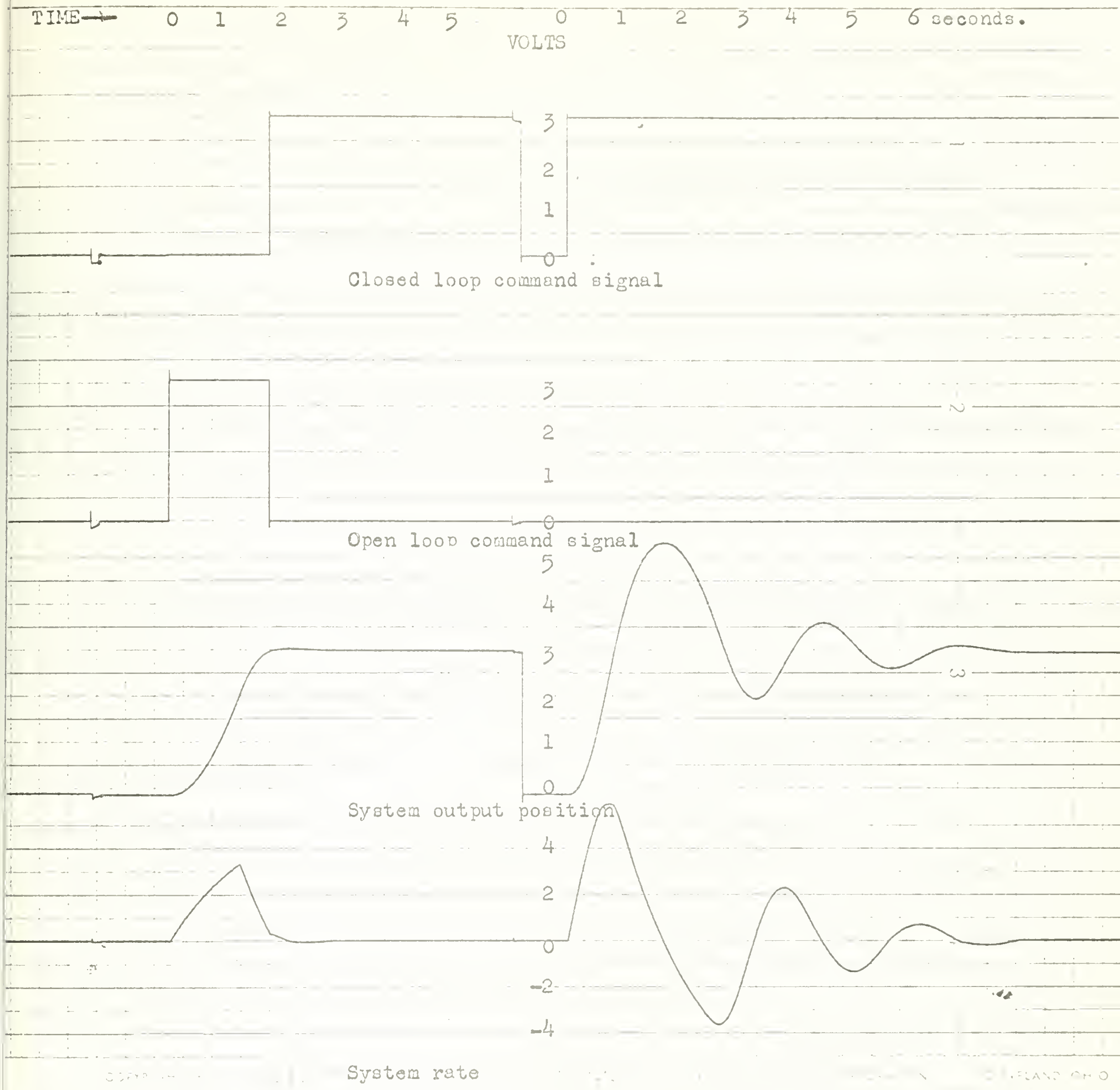


Fig. 15. Time invariant and closed loop response of DC servo system with a 3.05 volt input.

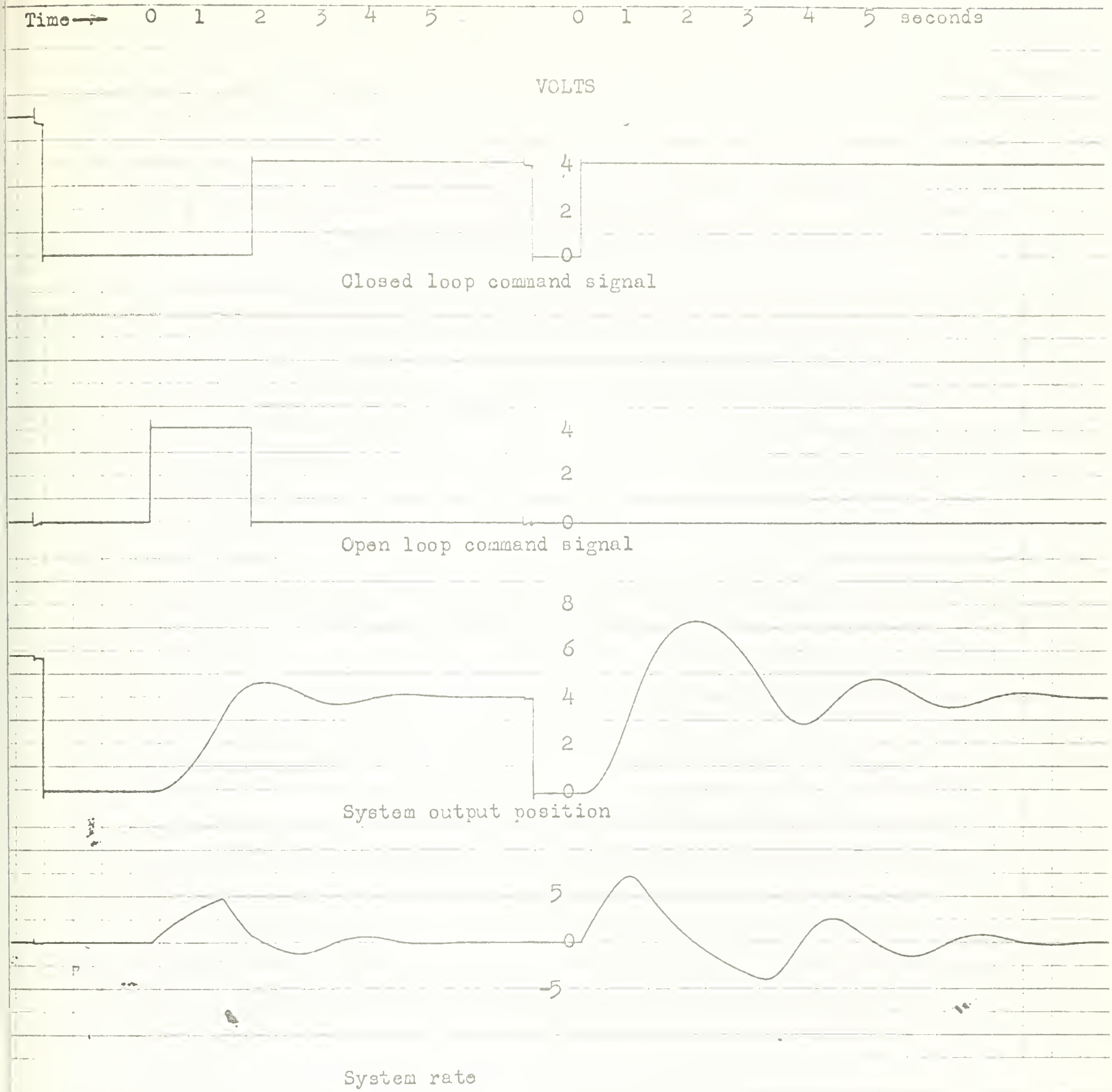


Fig. 16. Time invariant and closed lo p response of DC servo system with a 4.0 volt input.

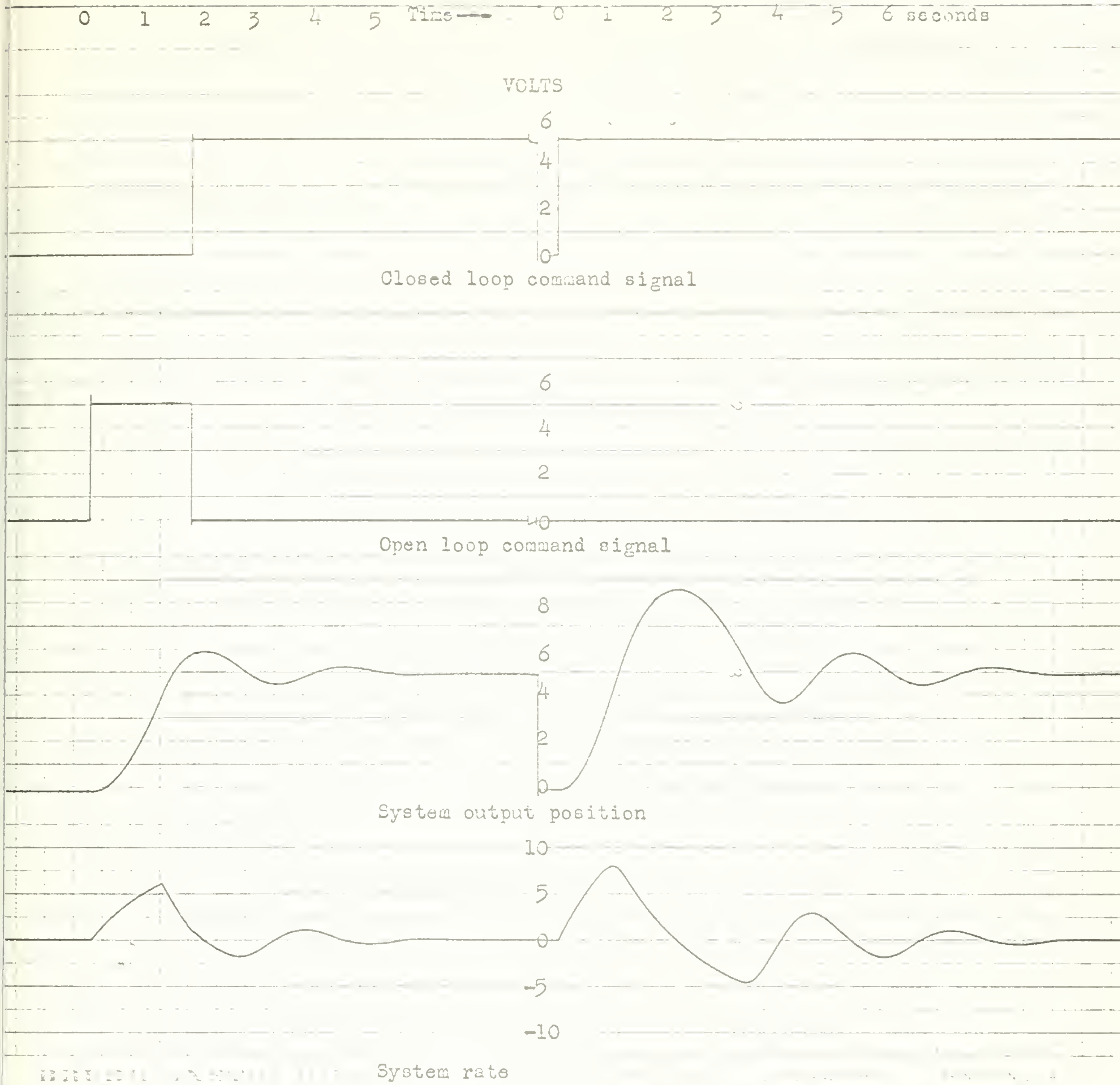


Fig. 17. Time invariant and closed loop response of DC servo system with a 5.0 volt input.

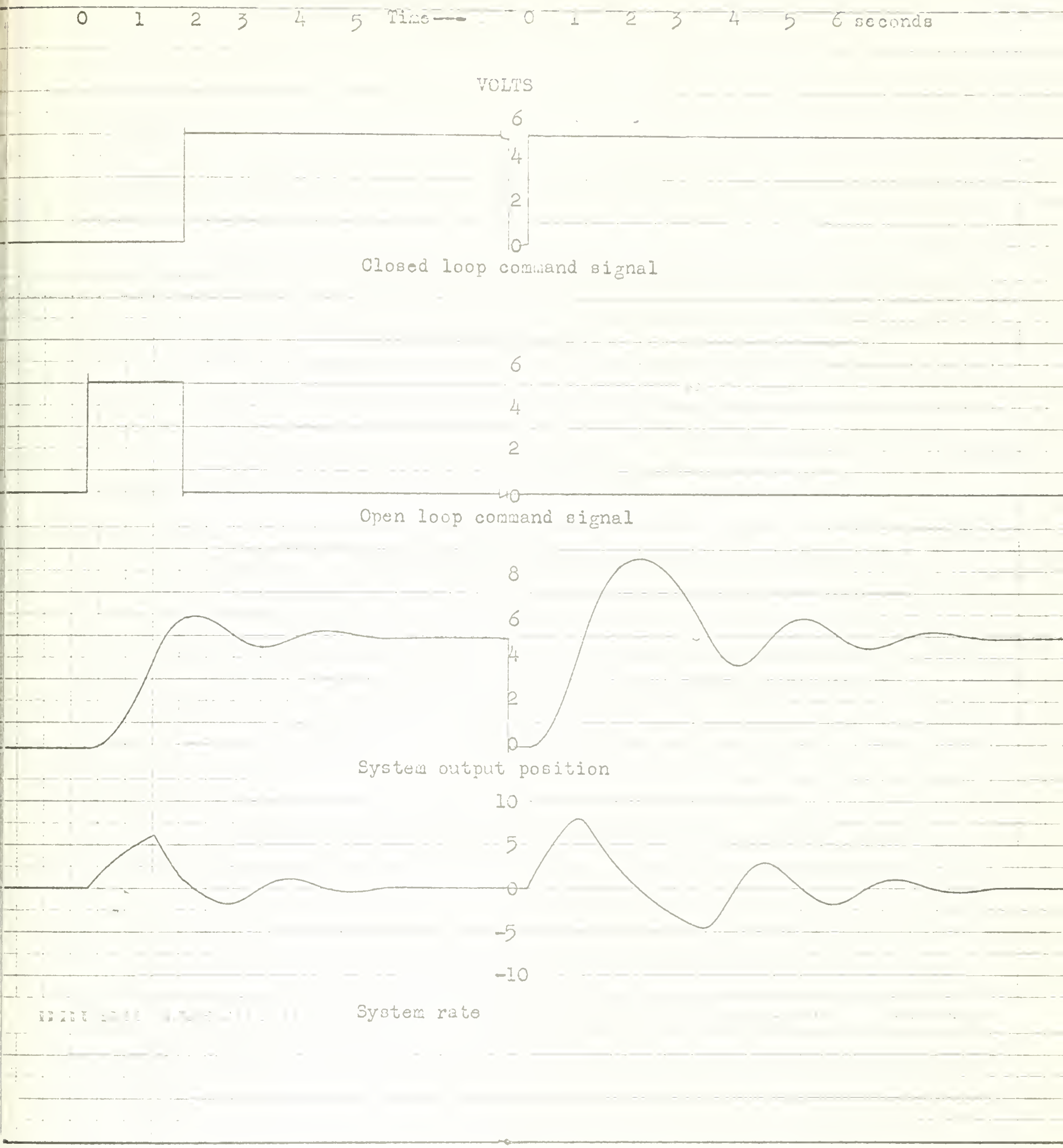


Fig. 17. Time invariant and closed loop response of DC servo system with a 5.0 volt input.

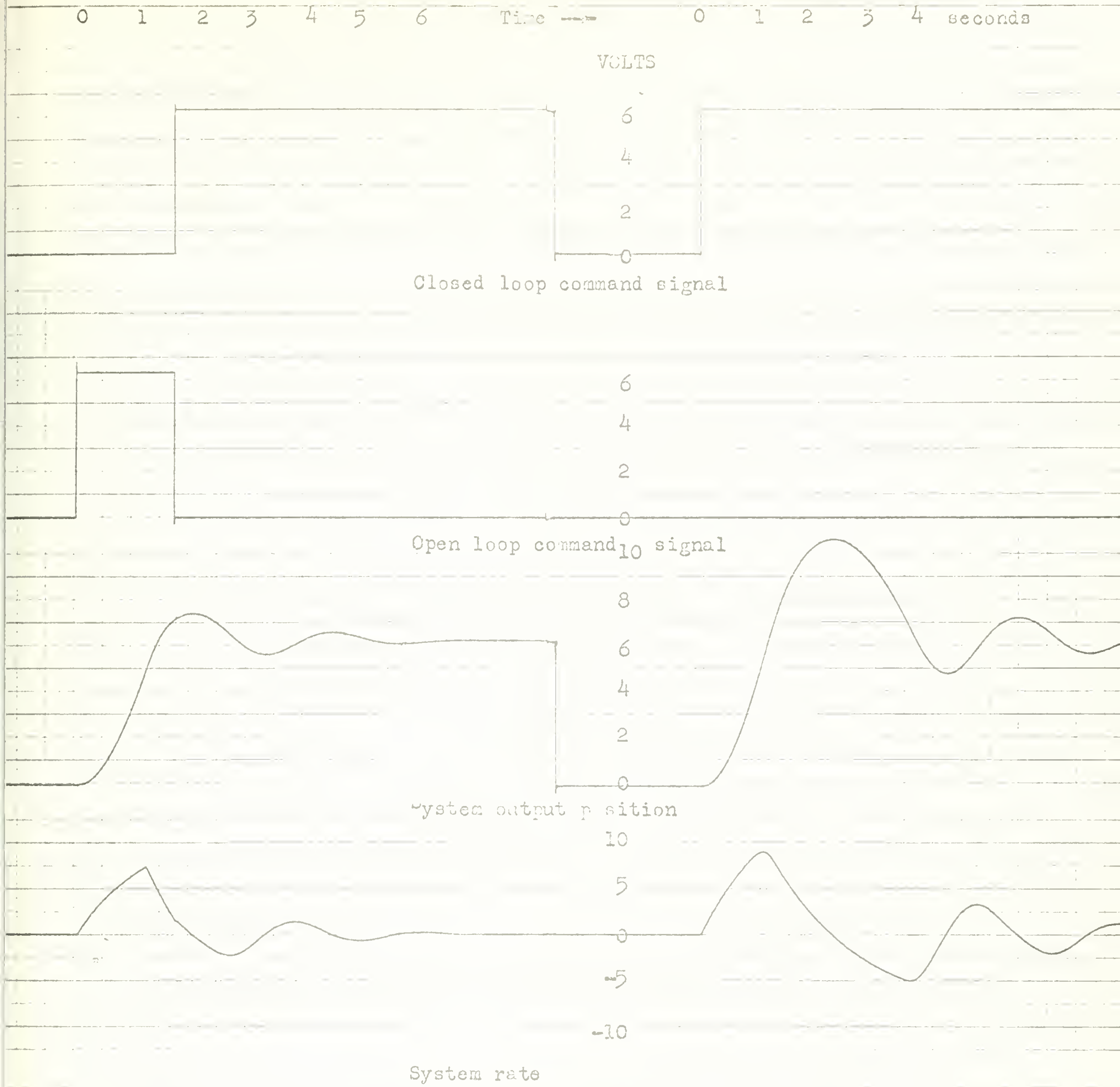


Fig. 18. Time invariant and closed loop response of DC servo system with a 6.3 volt input.

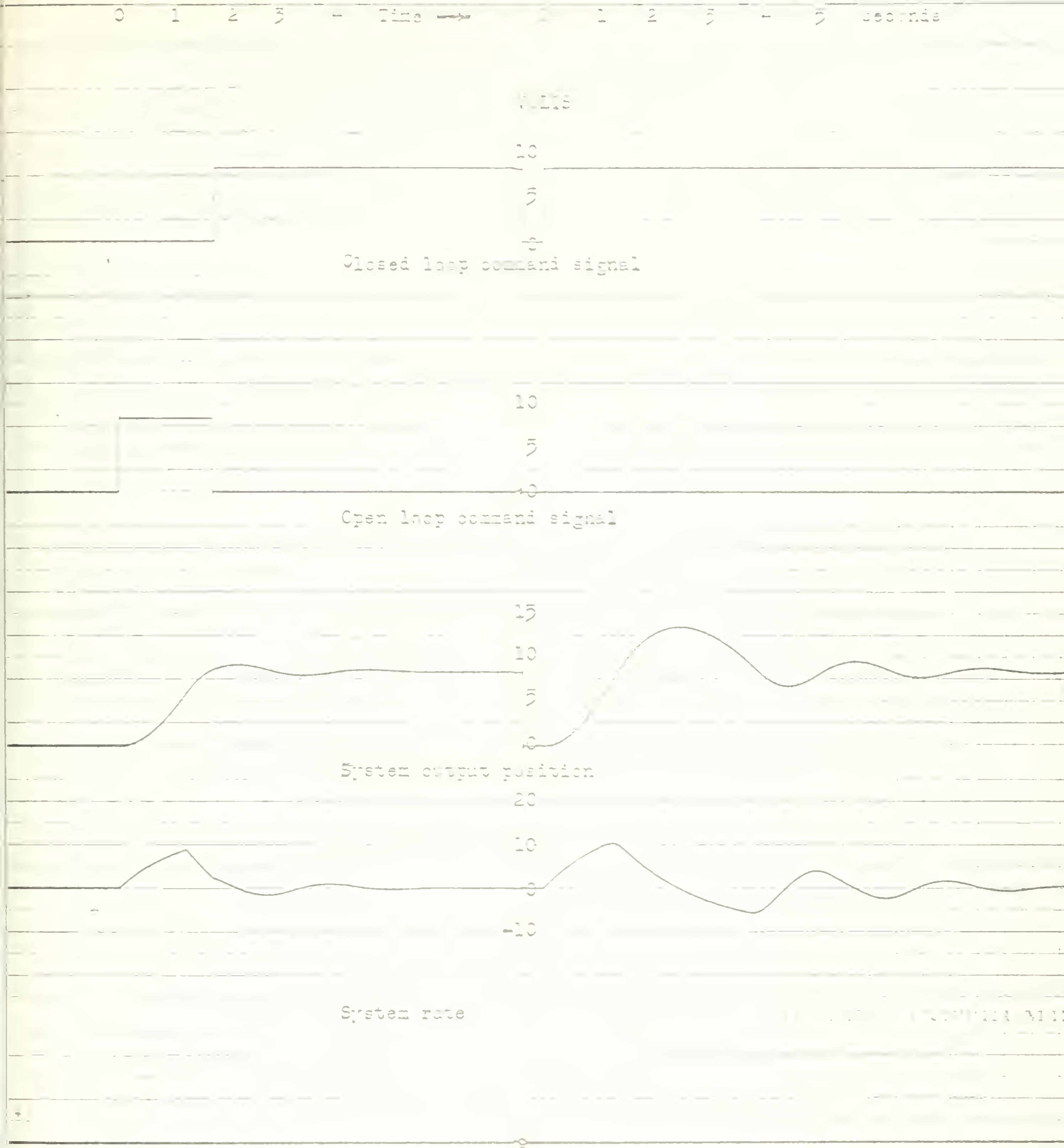


Fig. 19. Time invariant and closed loop response of DC servo system with an 8.- volt input.

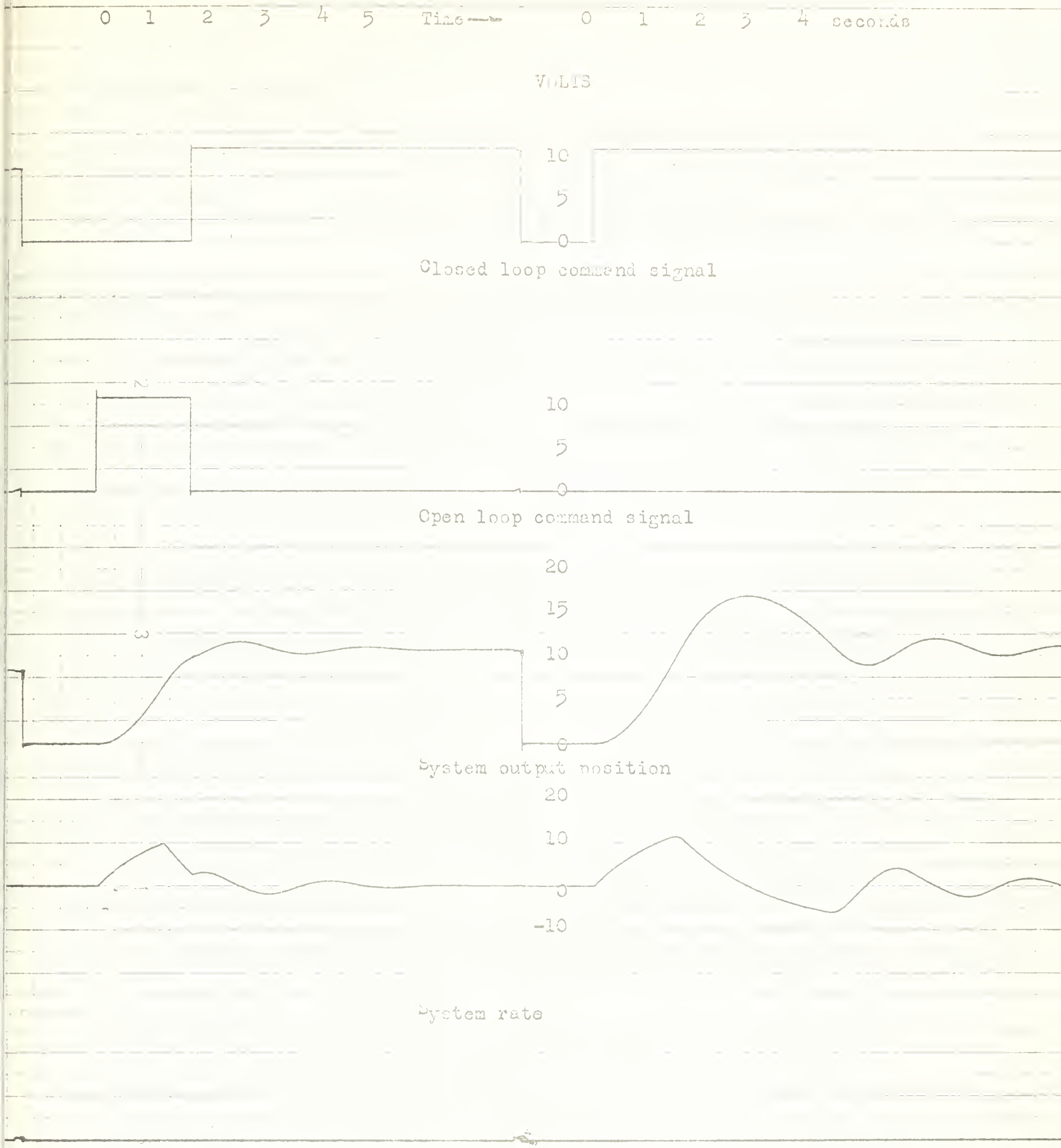


Fig. 20. Time invariant and closed loop response of DC servo system with an 10.5 volt input.

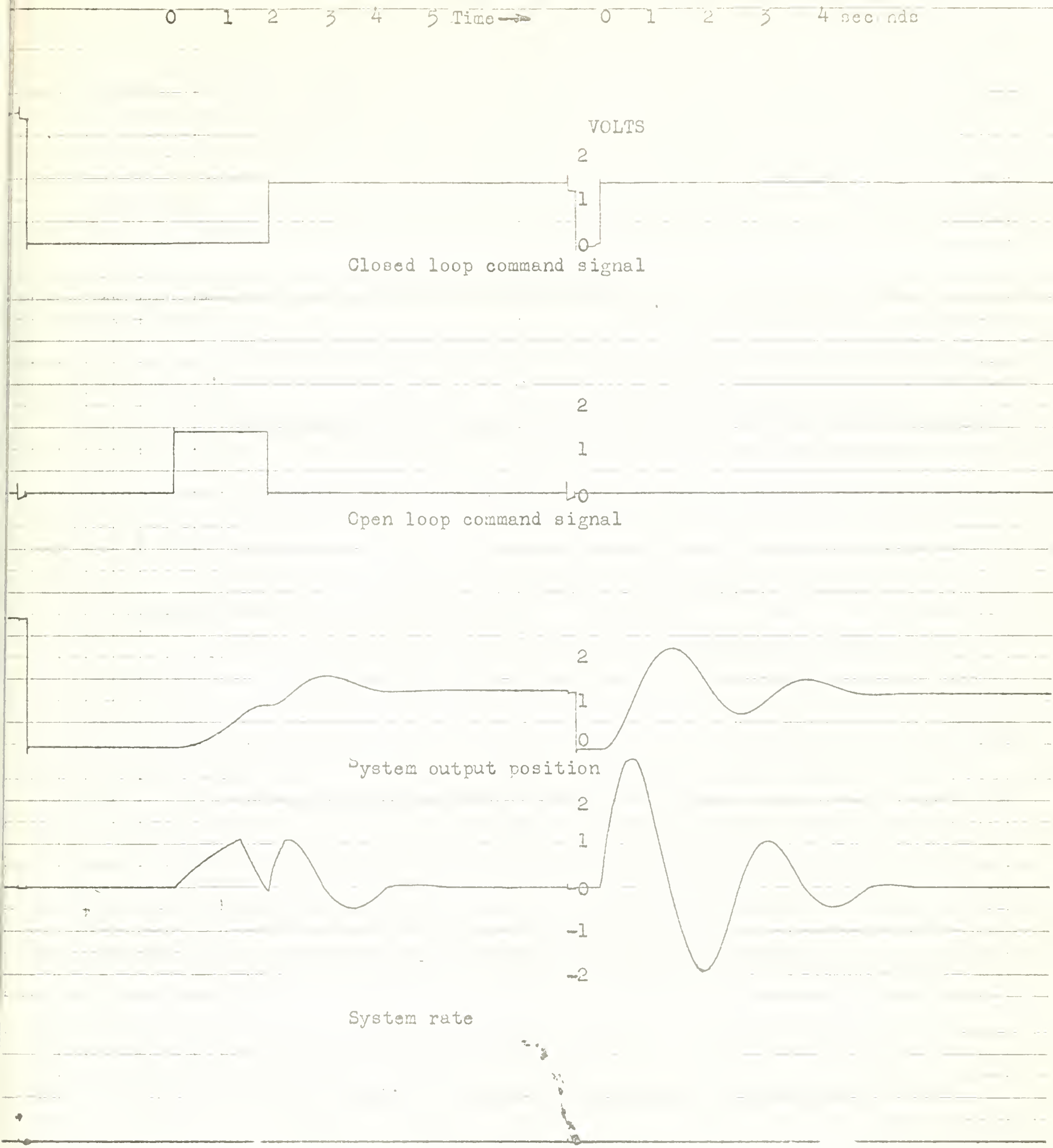


Fig. 21. Time invariant and closed loop response of DC servo system with a 1.4 volt input.

The controller reversed the input to the motor at the proper time but the reversal is not indicated on this trace. It was not possible to record the input to the motor (amplifier output) showing the reversal of driving voltage, because of grounding problems. A ground at the motor input would cause one or both of the power transistors to burn out in the power amplifier. The third trace is the system output and the fourth trace, the system rate. The time t_s, the time at which the polarity or sign of V is reversed, is readily observed on the last trace, system rate, as there is a definite change in slope at that time. Time on all the traces reads from left to right with t_o signified by the commencement of the open loop command signal and t_f by the end of this command and the start of the closed loop command signal.

It was mentioned earlier that an output signal of eight volts would saturate the power amplifier. It follows, then, that if $C \geq 4$, the amplifier will saturate when the driving voltage is reversed. The reason for this is that at the time of voltage reversal a back electromotive force has built up in the motor which approaches the driving voltage. This back emf has the effect of increasing current requirement from the amplifier when the driving voltage is reversed. Therefore, at the time of switching, the system should tend toward saturation if the signal voltage is greater

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It was mentioned earlier that an output signal of eight volts would saturate the power amplifier. It follows, then, that if $C \geq 4$, the amplifier will saturate when the driving voltage is reversed. The reason for this is that at the time of voltage reversal a back electromotive force has built up in the motor which approaches the driving voltage. This back emf has the effect of increasing current requirement from the amplifier when the driving voltage is reversed. Therefore, at the time of switching, the system should tend toward saturation if the signal voltage is greater

than four volts. This saturation caused a reduction in the braking or deceleration power available. From this it follows that the system output would tend to overshoot. These effects may be seen in Figures 16, 17, and 18. By analyzing the system rates at time t_s and t_t for these figures it can be demonstrated that saturation does occur in the time increment $t_s \leqq t \leqq t_t$. At time t_s, for the three figures mentioned above, the system rate increased proportionately with the command signal as called for by the theory. Yet at time t_t, the system rate was larger than predicted meaning that the system did not decelerate sufficiently which resulted in overshoot at time t_t. For example, the system rates at time t_s and t_t for the five volt input (Fig. 17) are 6.1 and .95 volts per second respectively. For an input of 6.2 volts (Fig. 18) one would predict by the principle of superposition that the rates would be increased by the factor 6.2/5.0 or would be 7.5 and 1.1 volts per second. Actually, Fig. 18 shows the rates to be 7.5 and 1.5 volts per second. The rate at time t_s agrees with the theory but at time t_t the rate differs by .4 volts per second. This difference can be accounted for by the saturation that occurs when the voltage is reversed.

When the system input exceeds eight volts, saturation is predicted at t_o and at t_s. One would expect, therefore, that the system output would never reach the

desired point at time t_t or in other words, the system will have undershoot at time t_t. The reason for this phenomenon is that the driving voltage is never large enough to provide the response called for by the theoretical equations. The results of this type of saturation are shown in Figures 19 and 20. Fig. 20 clearly demonstrates the effect on system output.

Fig. 21 shows the response of the system with a small input, 1.4 volts. Here the system output, by time t_t, does not reach the desired value. The primary cause for this is that the small driving voltage is of insufficient magnitude to overcome the stiction and friction in the system.

From these experimental tests it can be seen that the time invariant switching scheme does provide deadbeat response over a range of inputs. If deadbeat response is desired in a positioning feedback control system, the method of time invariant switching can provide this in a given time increment over a designed range of inputs.

6. Test of the time invariant controller applied to an amplidyne driven motor.

The time invariant controller has been shown to work over a range of inputs with a quasi linear, second order system. One might now inquire as to its usefulness with a system that is quite nonlinear in that it exhibits hysteresis, excessive stiction, and saturation. To investigate this, a system was constructed that incorporated an amplidyne, a $\frac{1}{4}$ HP d-c shunt wound motor, and a permanent magnet, D. C. generator. The output of the generator, or tachometer, was integrated to simulate the system output. As in the previous experiment the system output and input was in volts. Fig. 22 depicts the system as set up in the laboratory with the relays performing the same functions as outlined in section 5. Fig. 23 is a picture of the system as it appeared in the laboratory.

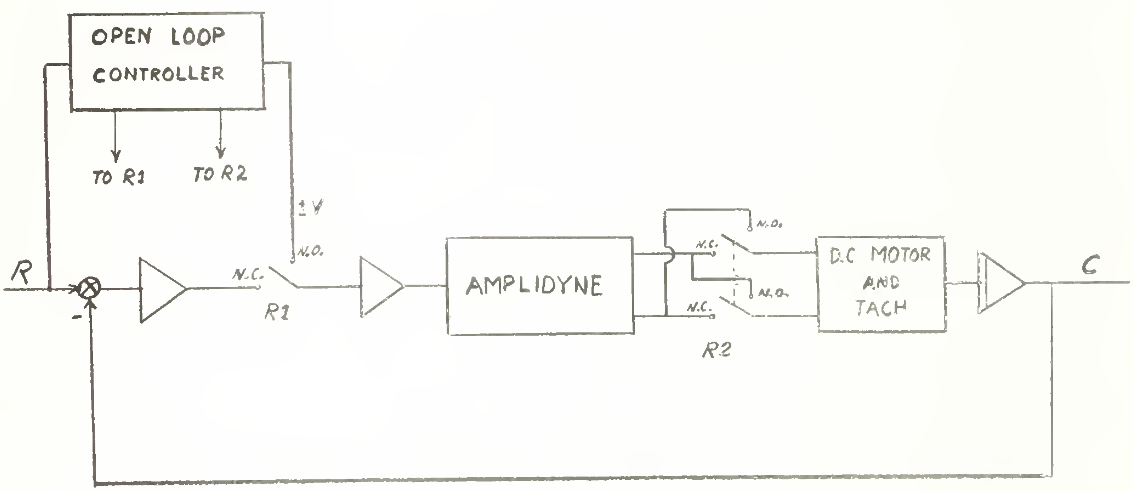


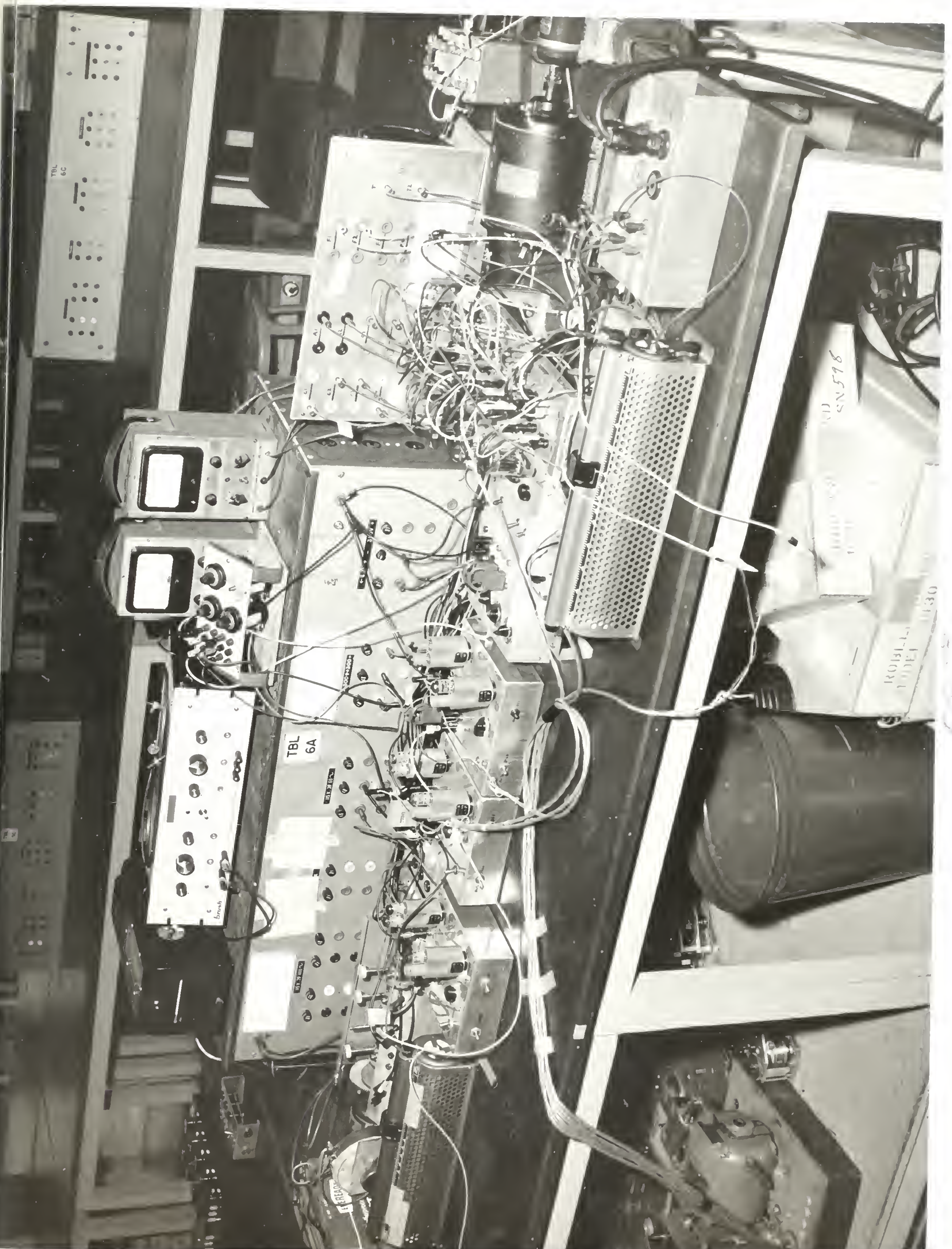
Fig. 22 Amplidyne driven motor servo system with time invariant controller.

A plot of the voltage input versus the tachometer voltage output for the open loop system appears on Fig. 24. Notice that at least two volts are required to start the system in motion and that at about six volts the system starts to go into saturation. The hysteresis loop is rather broad which could provide difficulty in system performance. From open and closed loop tests of the system, without the time invariant switching, the open loop transfer function was determined to be approximately:

$$\frac{110}{S(S+7)} \tag{17}$$

Depending on the test being conducted, the system gain varied from 100 to 115 and the reciprocal time constant from 6.9 to 7.5 inverse seconds.

Using the values of gain and the time constant from equation (17) and having a unity relationship between the open loop output and open loop driving voltage, t_s was determined from equation (14) to be about .33 seconds. Then t_r, from equation (12a), was evaluated as .27 seconds. The system was run with an input step of four volts. The response was not deadbeat so the times of switching were adjusted to yield a near deadbeat response. The timing circuit controls were not fine enough to permit precise adjustments of the switching times. The adjusted times were then measured and found to be .38 second for time t_s and .09 second for t_r.


TBL
6A
brush

T... Invariant-Servo Positioning System. Photographed for the thesis of Kenneth C. Malley.

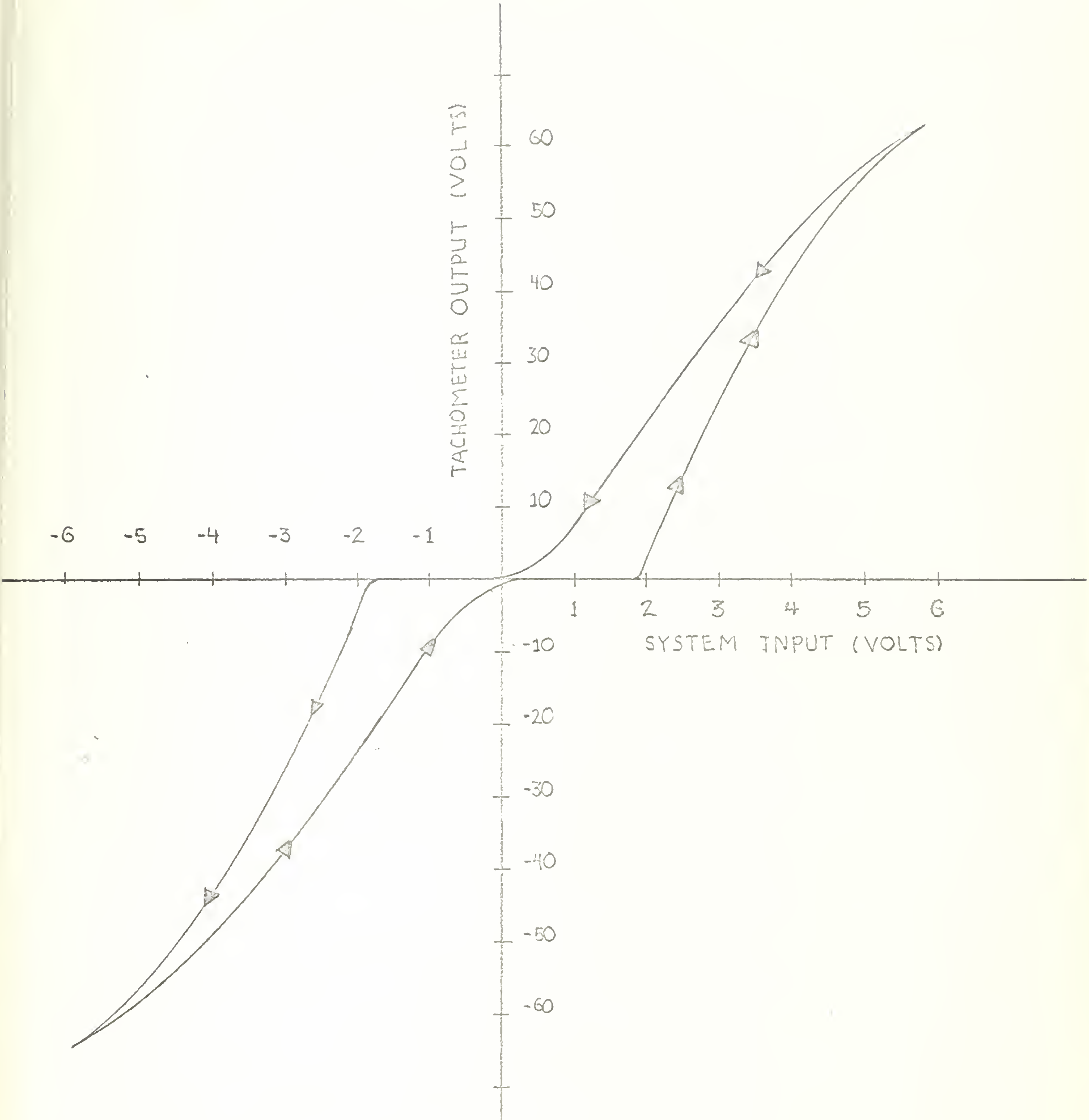


Fig. 24. Hysteresis loop of amplidyne driven servo system.

The acceleration time was .05 second longer than the computed value and the deceleration time was .18 second shorter than the computed value. This shorter deceleration time is probably due to the large friction that was present in the motor.

A Brush recorder was used to record the values of the closed loop command signal, the open loop command signal, the system output, the system rate, and the voltage applied to the motor. These recordings are shown in Figures 25 through 38. Following each time-invariant run to a given input, the system was run in the normal closed loop mode with the same quantities being recorded. A figure of the closed loop recording follows each time invariant figure. The maximum overshoot and time-to-steady-state was evaluated for all runs and a comparison of the times to steady state was made. The results of this evaluation are tabulated in Table II.

TABLE II

Amplidyne Servo	Open-closed loop		Closed loop		
Input	M_{pt}[1]	t_{ss}[2]	M_{pt}	t_{ss}	Δt_{ss}[3]
3.0 volts	1.0	1.18 sec.	1.0	.59 sec.	-.59 sec.
3.5	1.0	1.00	1.0	.50	-.50
4.0	1.0	.50	1.26	1.00	.50
4.5	1.0	.48	1.31	1.05	.57
5.0	1.0	.48	1.58	1.00	.52
5.5	1.25	1.03	2.02	.93	-.10
6.0	1.88	1.00	2.12	1.30	.3

Note:

1. M_{pt}: Ratio of maximum output to desired output.

2. Time to steady state operation, taken to be when system is within .1 volts of command signal.

3. Δt_{ss} is defined as the difference between the times to steady state for the open-closed loop and closed loop operation.

From the recordings it can be seen that the range from four to five volts the response was almost dead-beat. (Figures 25 through 30.) When the input voltage was in excess of five volts the system output started to overshoot the desired value at time t_t. As in

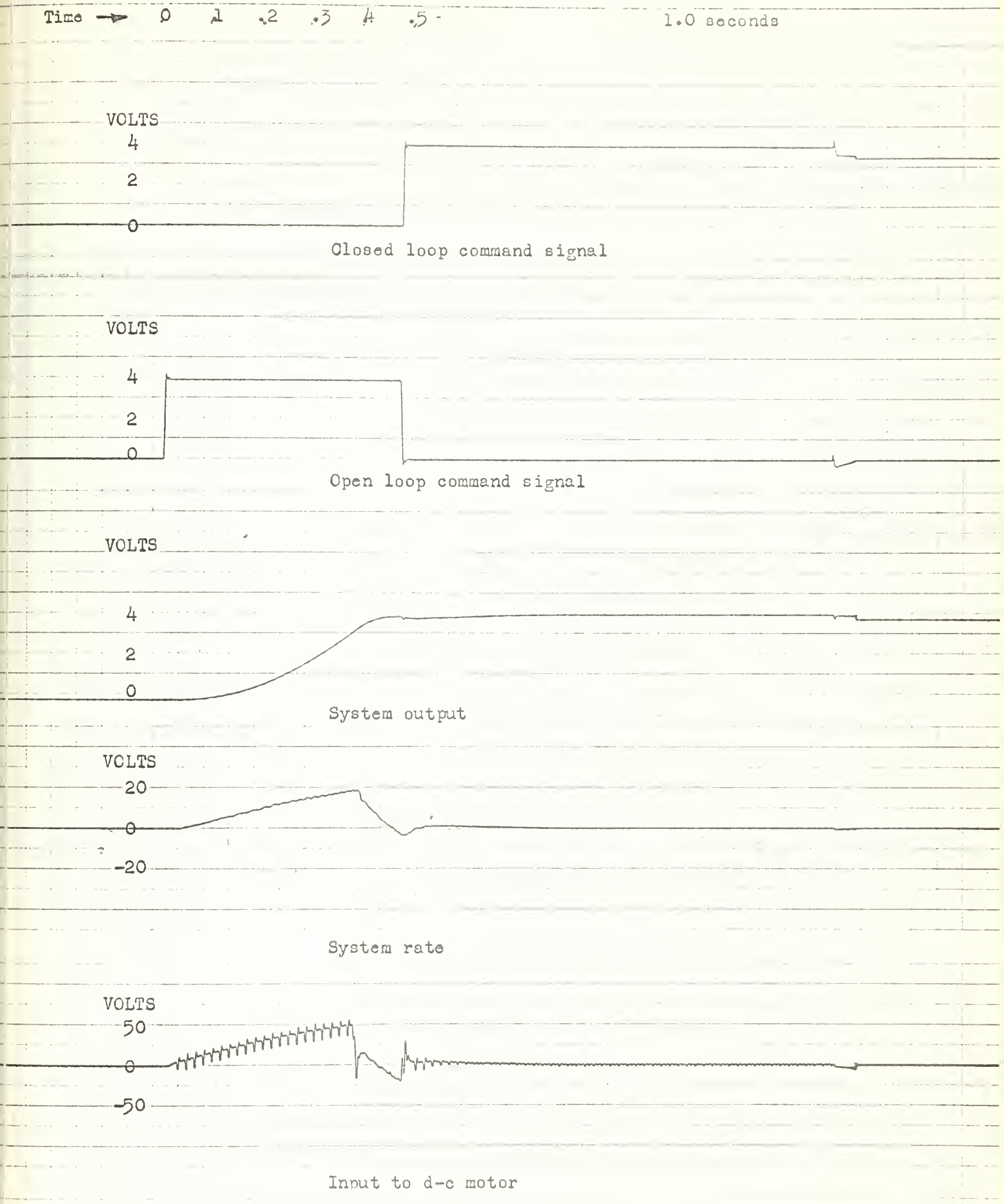


Fig. 25. Time invariant response of amplidyne driven motor servo system with a four volt input.

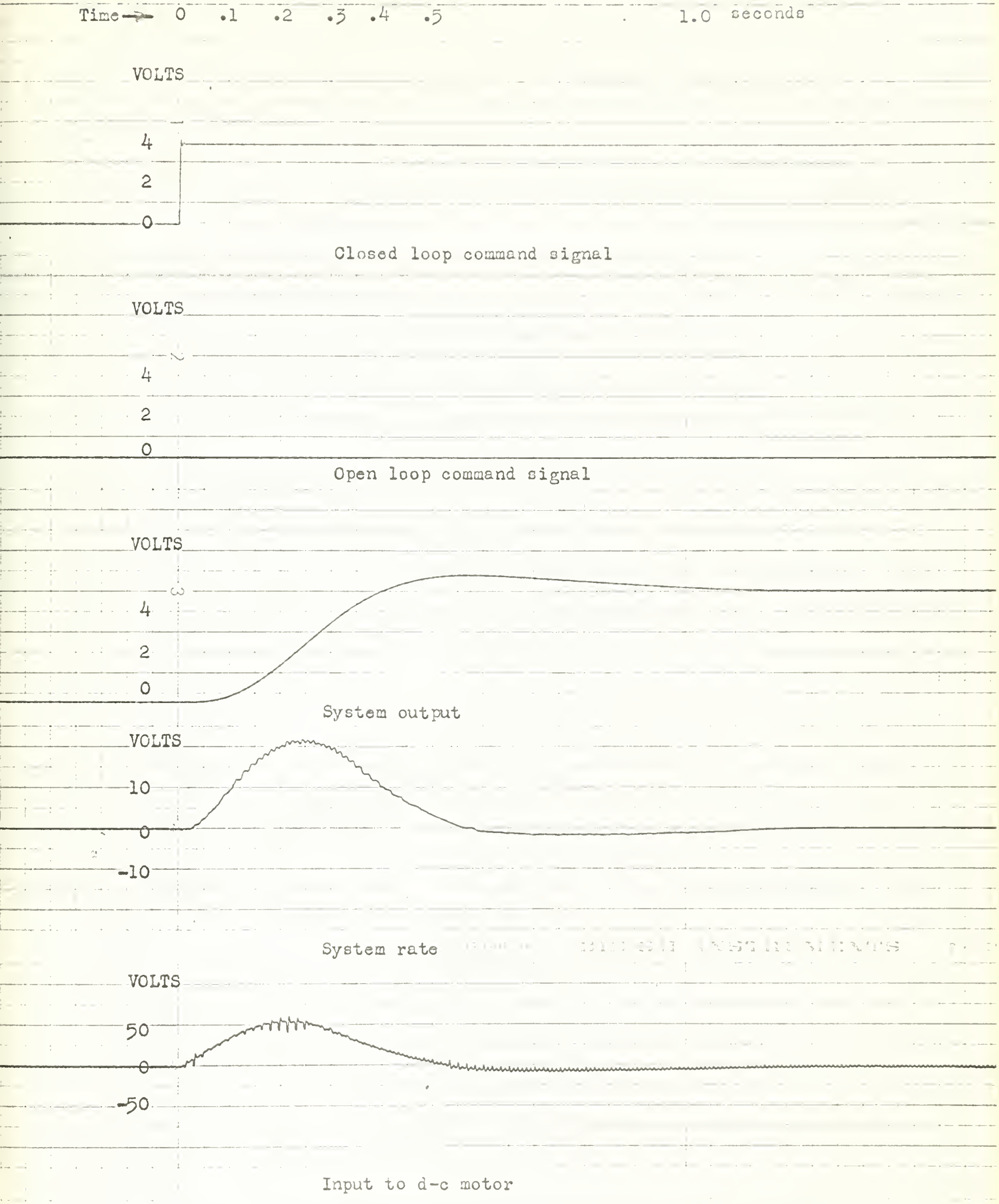


Fig. 26. Closed loop response of amplidyne driven motor servo system with a four volt input.

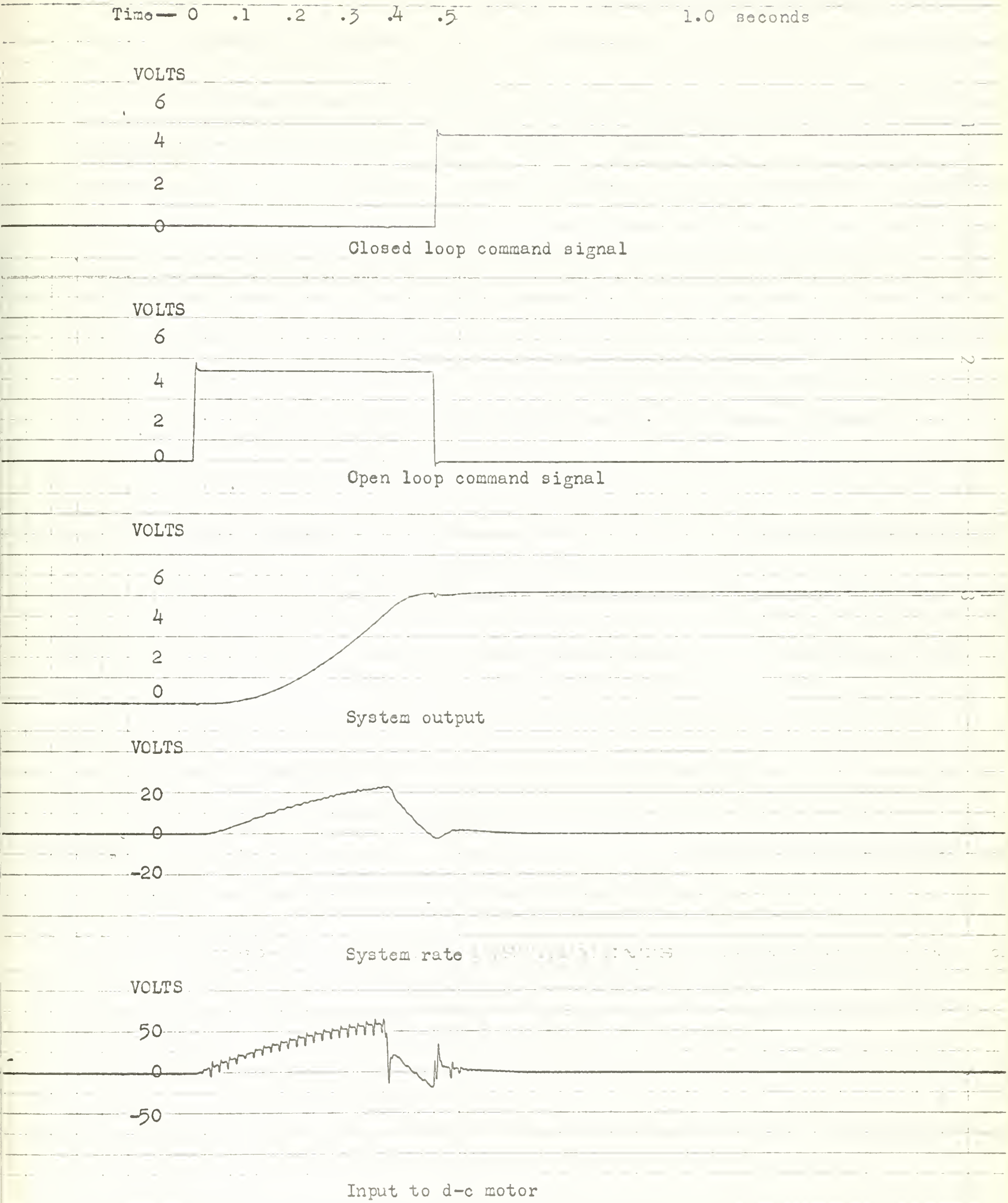


Fig. 27. Time invariant response of amplidyne driven motor servo system with a $4\frac{1}{2}$ volt input.

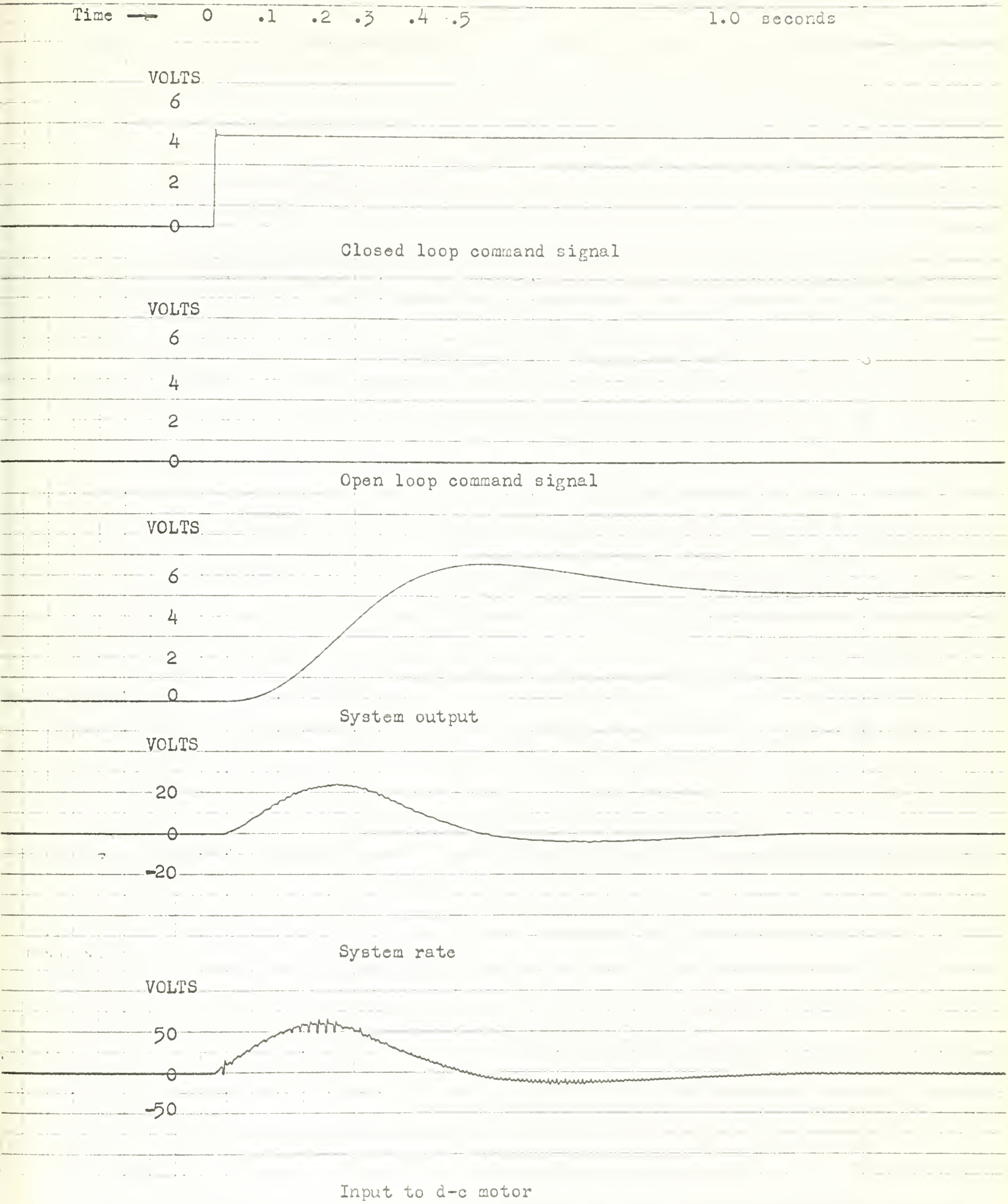


Fig. 28. Closed loop response of amplidyne driven motor servo system with a $4\frac{1}{2}$ volt input.

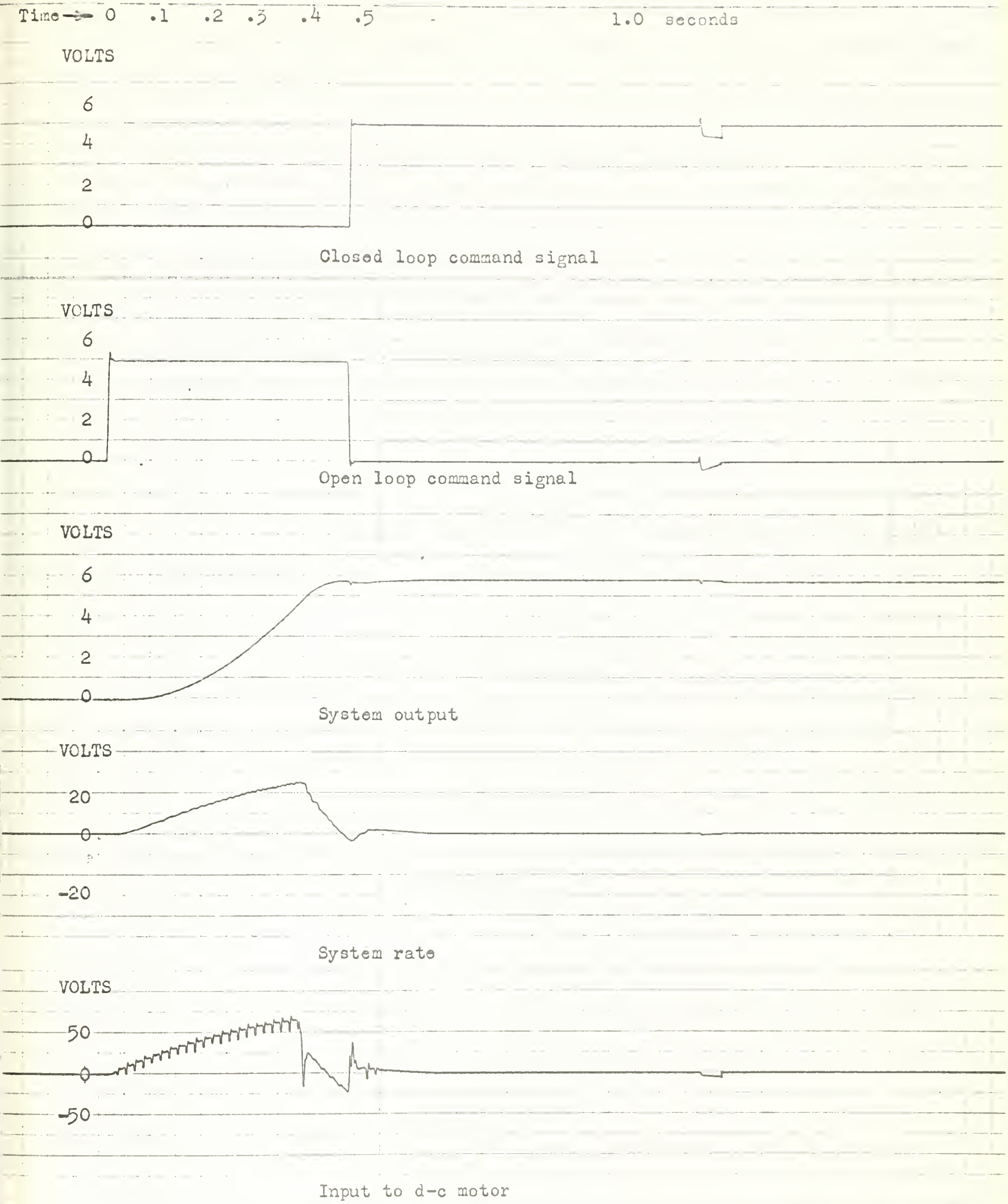


Fig. 29. Time invariant response of amplidyne driven motor servo system with a five volt input.

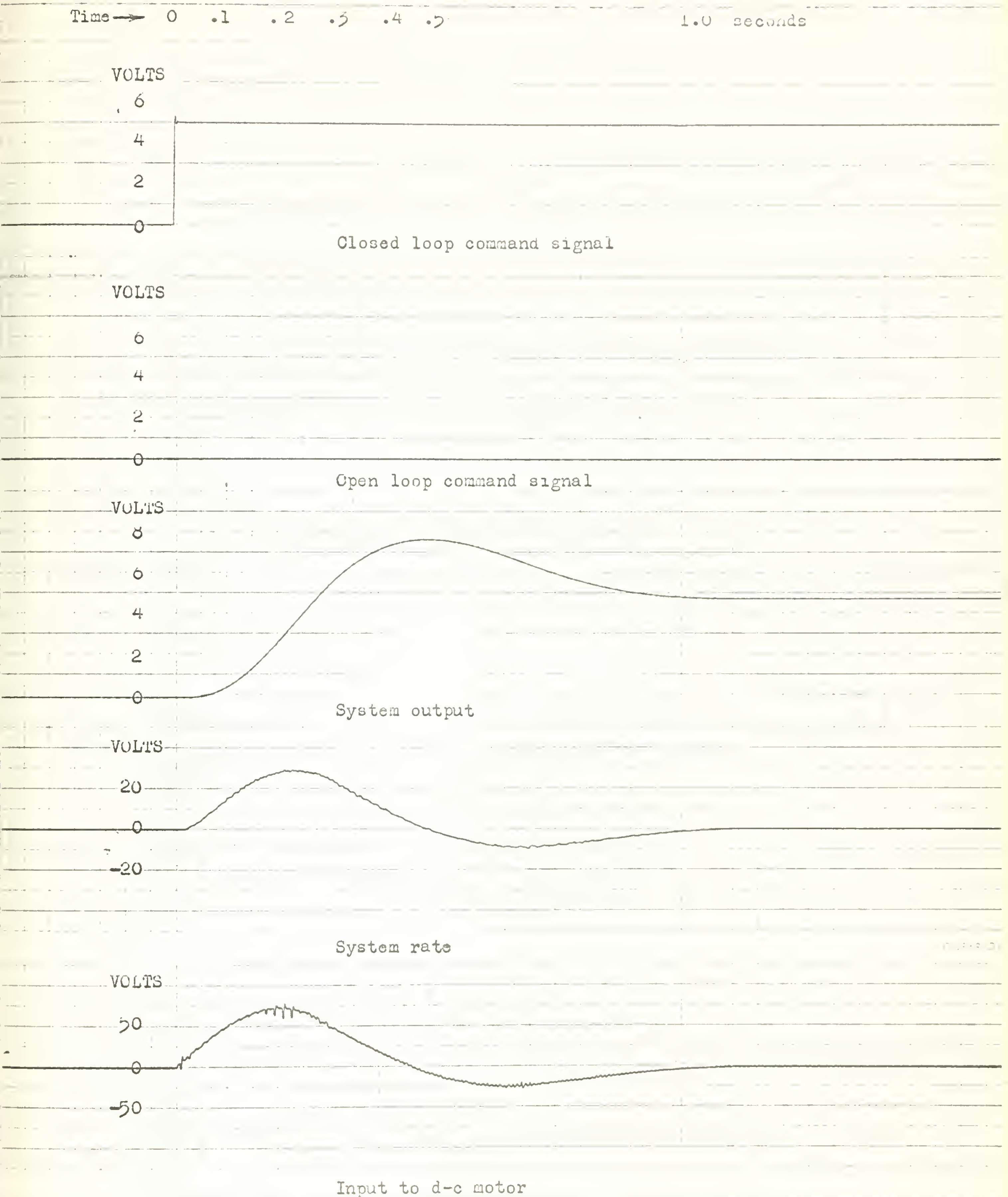


Fig. 30. Closed loop response of amplidyne driven motor servo system with a five volt input.

section 5 with the D. C. servo, this is due to saturation of the amplidyne when the driving voltage is reversed. This type of response is shown in Figures 31 through 34.

For step inputs less than four volts system performance deteriorated rapidly. This was due almost entirely to the excessive stiction and friction in the system. It was found that the system did not respond to a command less than two volts or about 30% of the operating range. The system response to inputs of 3 and 3.5 volts are shown in Figures 35 through 38.

As might be expected, the use of an open-loop controller with a far-from-ideal plant does not lead to acceptable performance.

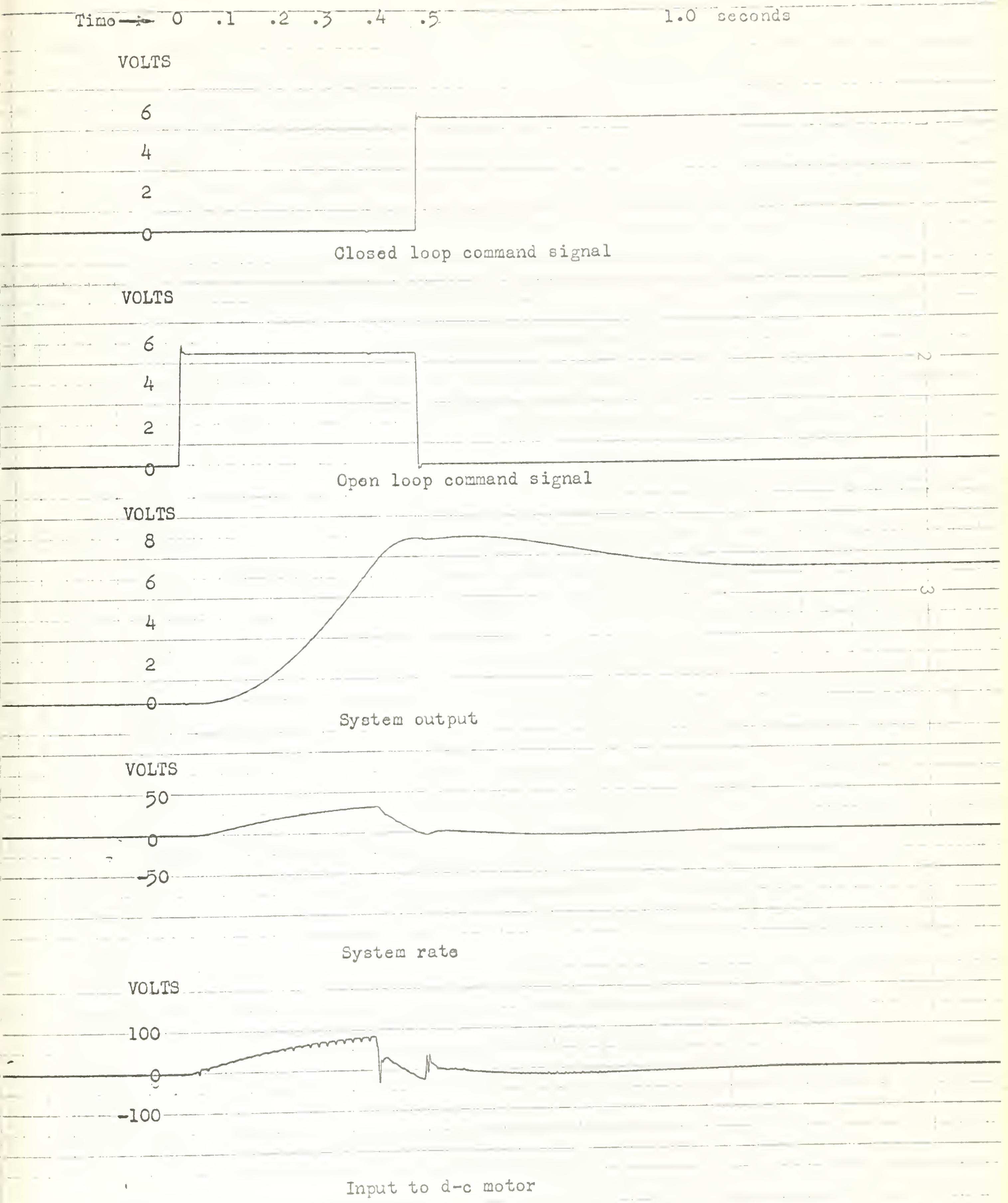


Fig. 31. Time invariant response of amplidyne driven motor servo system with a $5\frac{1}{2}$ volt input.

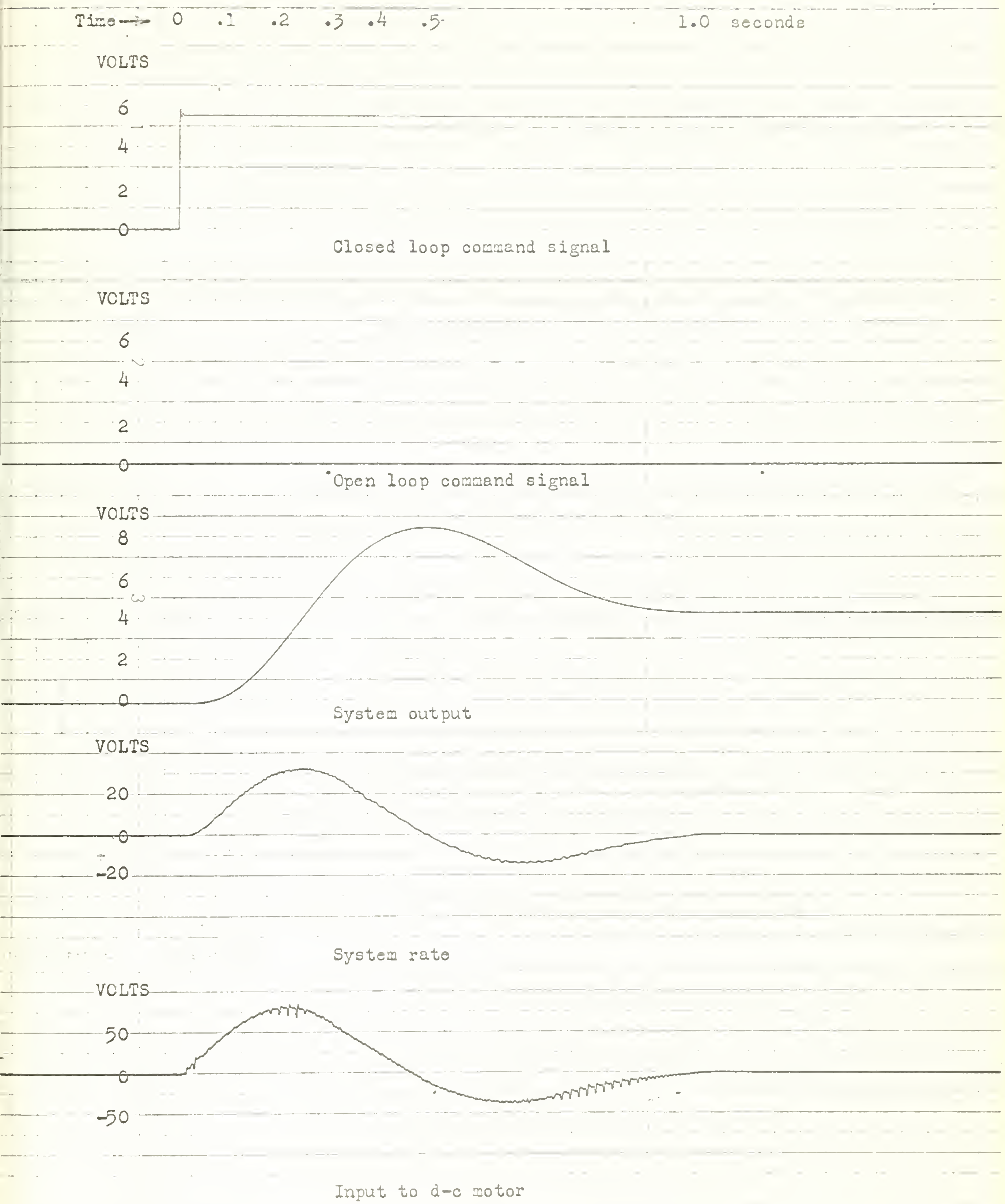


Fig. 32. Closed loop response of amplidyne driven motor servo system with a $5\frac{1}{2}$ volt input.

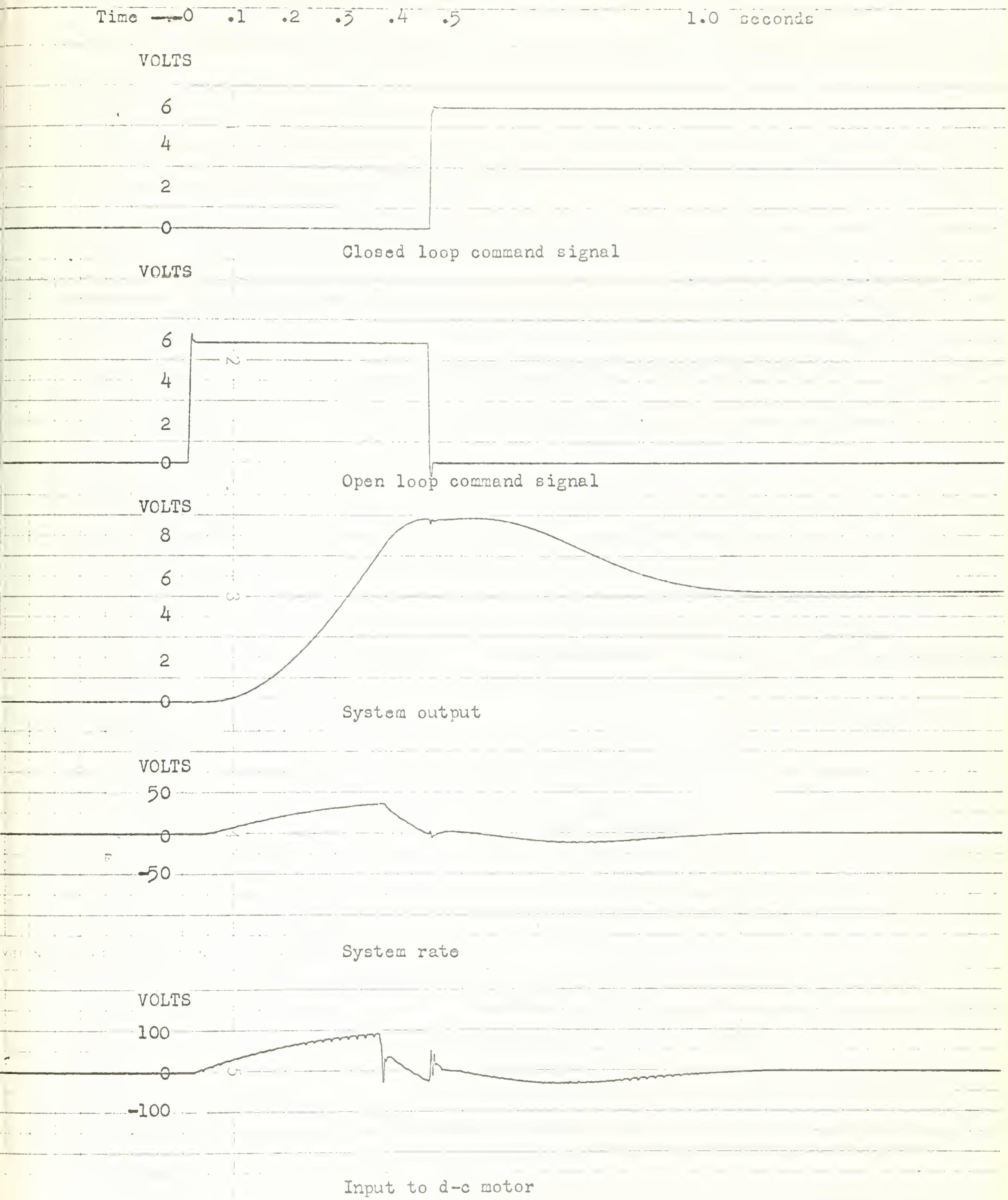


Fig. 33. Time invariant response of amplidyne driven servo system with a six volt input.

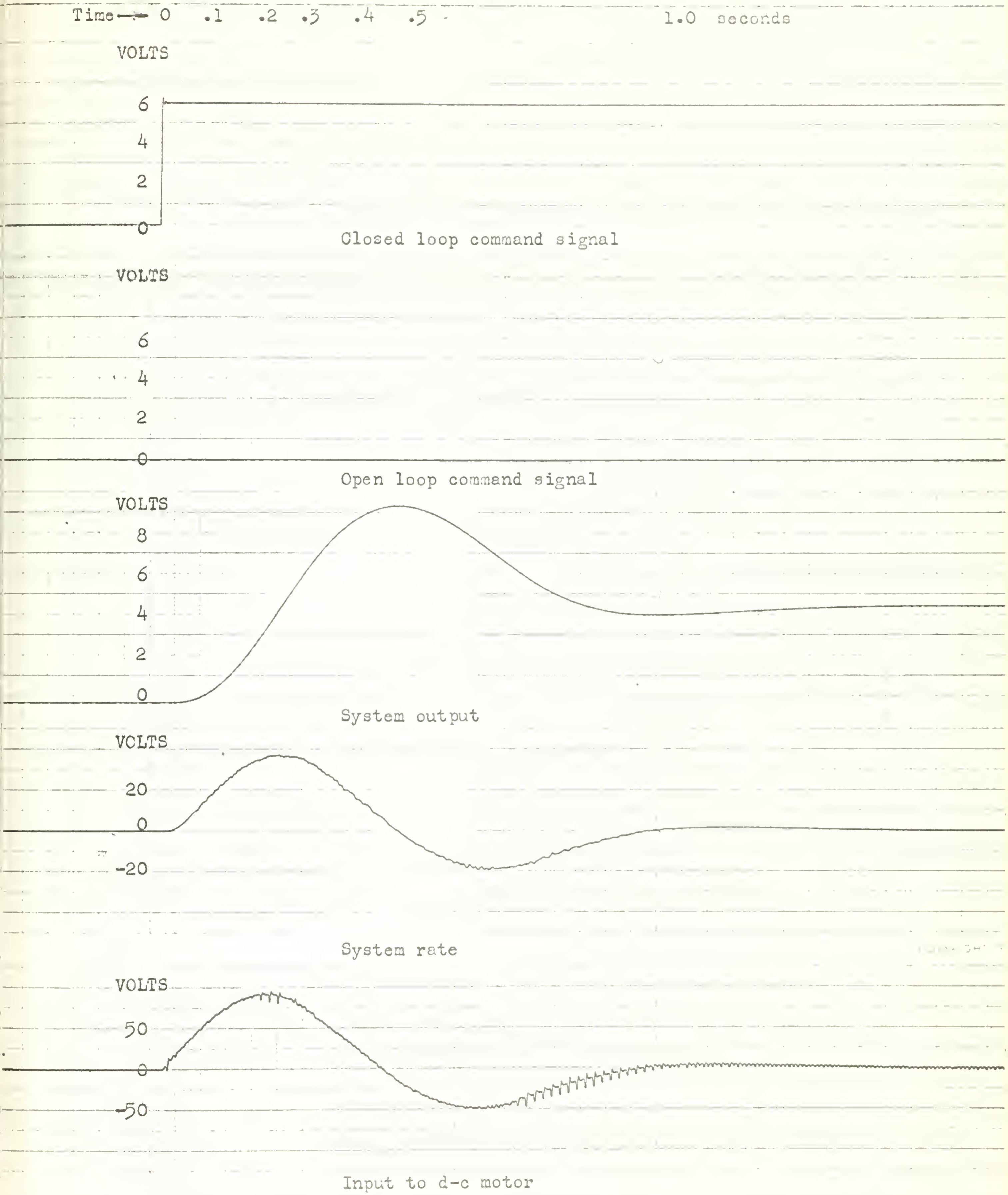


Fig. 34. Closed loop response of amplidyne driven servo system with a six volt input.

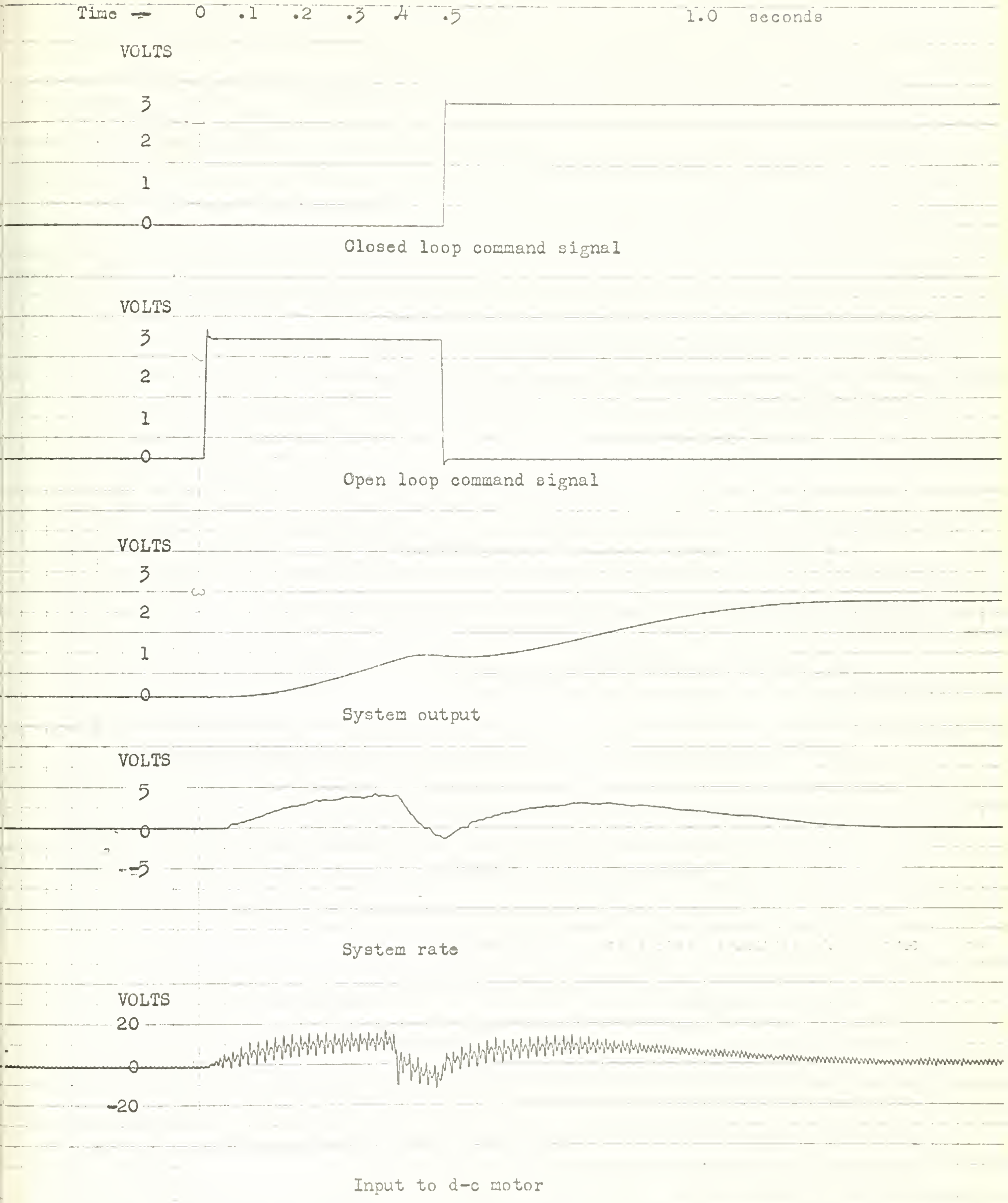


Fig. 35. Time invariant response of amplidyne driven motor servo system with three volts input.

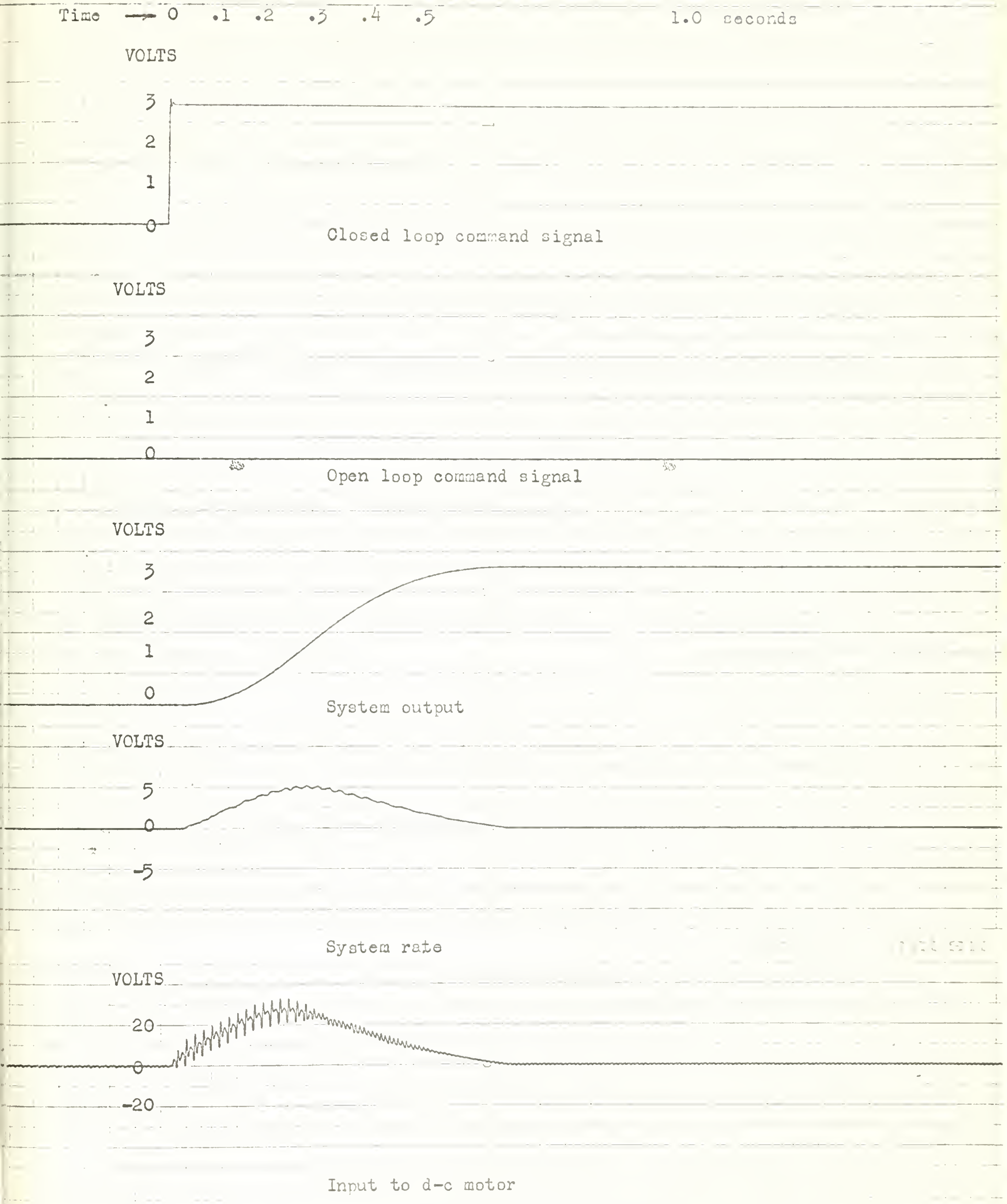


Fig. 36. Closed loop response of amplidyne driven motor servo system with a three volt input.

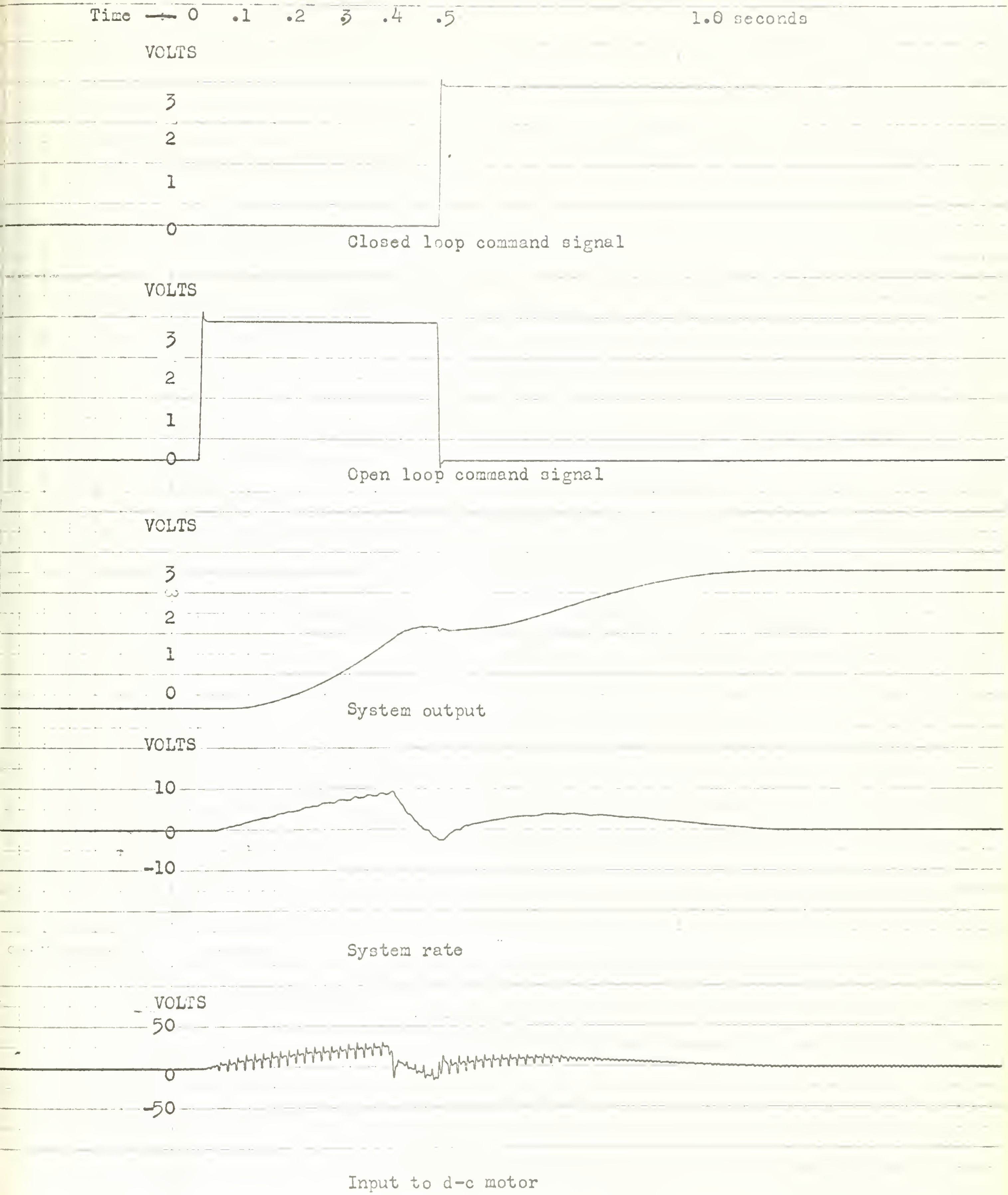


Fig. 37. Time invariant response to amplidyne driven motor servo system with a $3\frac{1}{2}$ volt input.

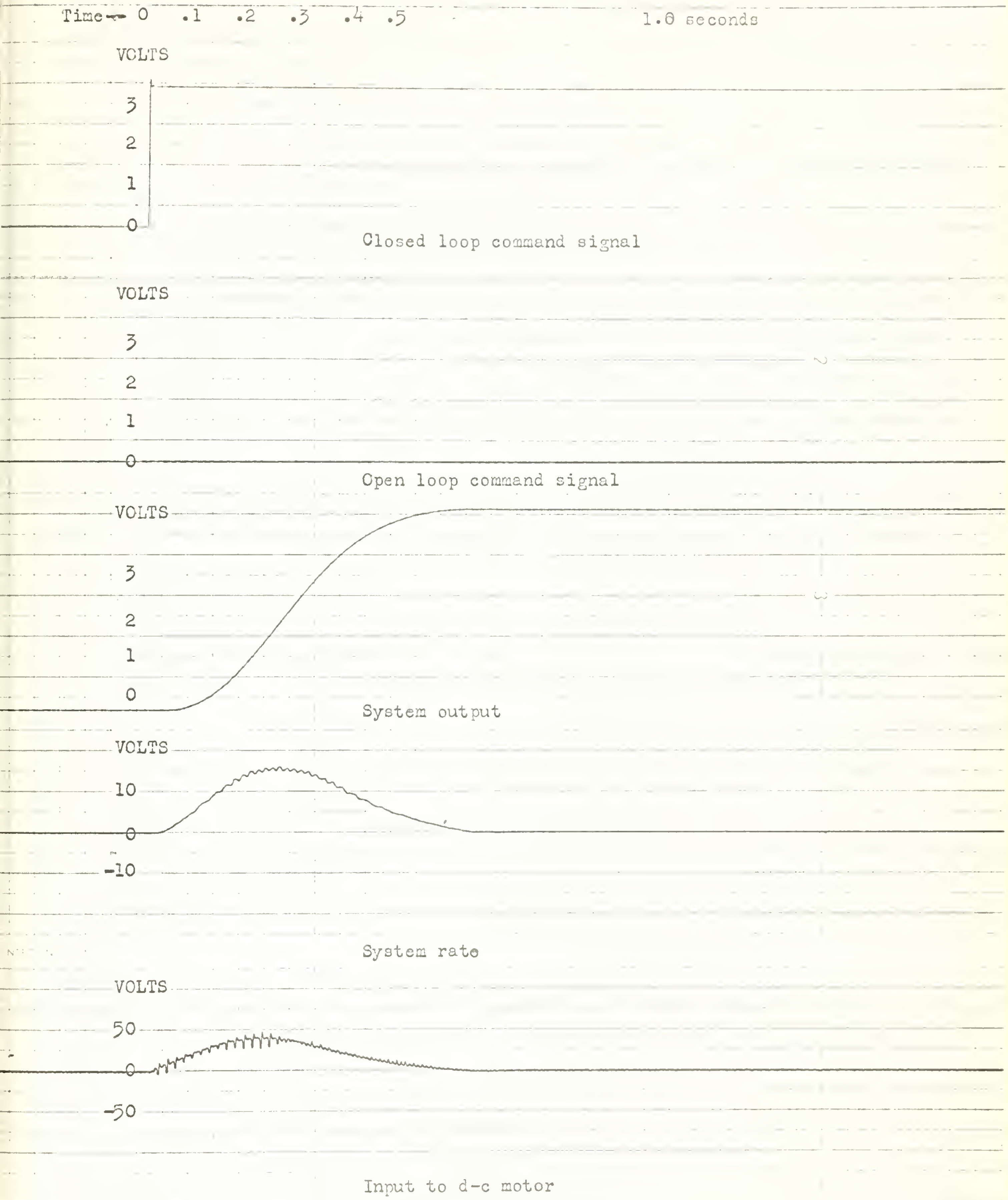


Fig. 38. Closed loop response of amplidyne driven motor servo system with a $3\frac{1}{2}$ volt input.

7. Conclusions.

In the search for a simple scheme to provide deadbeat response for step inputs to a second order system, attention was concentrated on an open-closed loop controller. For the system investigated, switching time in the open loop mode were fixed and the output from the controller to the plant was proportional to the magnitude of the step input.

Using this scheme, any linear system can be controlled with deadbeat response (Principle of superposition). To control the second order plant considered in this study only one switching point is required. The experimental program was intended to suggest the shortcomings of this approach when applied to a relay system with its non-linearities and higher order dynamics.

The first real system studied was relatively free from any non-linearities and provided near deadbeat response over approximately 50% of the designed operating range. A system with rather pronounced non-linearities was also chosen for study and provided near deadbeat response over less than 20% of the desired operating range. It is concluded, then, that if deadbeat or near deadbeat response is desired, the time invariant controller investigated in this thesis can be used to force this response in a linear system . With a non-linear system, the controller is of little use. In any event, the time of response to any size step requires the same interval of time for open loop operation.

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4. S. I. Leberman, A bang-bang attitude control system for space vehicles, Aerospace Engineering, October, 1962.

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6. G. J. Thaler, M. P. Pastel, Analysis and design of Nonlinear Feedback Control Systems. Chapter 7.

7. "Program Analog" was written by Dr. J. R. Ward of the U. S. Naval Postgraduate School. The program as used is shown in Appendix I.

8. M. E. Van Valkenburg, Network Analysis, Prentic-Hall, Inc., PP. 79-80.

9. D. D. McCracken, A guide in FORTRAN programming, John Wiley & Sons, Inc., New York, London , 1961.

APPENDIX I

PROGRAM ANALOG

A Control Data Corporation CDC 1604 digital computer is available at the U. S. Naval Postgraduate School. Dr. J. R. Ward of the Postgraduate School, Department of Electrical Engineering, has written a FORTRAN[8] language program for this computer which simulates an analog computer. The program is referred to as Program Analog. It retains all the versatility of the analog computer and provides both a graphical and numerical output. With this program many of the problems of magnitude and time scaling as well as the difficulties in simulating nonlinearities associated with an analog computer are avoided. The program, which is reproduced in Figures 39 through 41, uses a modified Runge-Kutta integration scheme.

This program was used for the investigations carried out in section 2. Fig. 39 is the main part of the program. It consists of the necessary instructions for the analog simulation of a given problem and does not require any changes as the problem is varied. This part of the program may be thought of as the analog computer. Fig. 40 is the FORTRAN simulation of the

* * * * * * * *

[9]D. D. McCracken, A guide in FORTRAN programming, John Wiley & Sons, Inc., New York, London, 1961.

problem board. This part of the program must be changed in the same manner as one would change the wiring on a problem board. Fig. 41, the data input, sets the initial conditions, potentiometer settings, problem time and provided means by which various output data can be obtained. A typical numerical output for a Program Analog problem appears in Fig. 42. Figures 3 through 10 in the body of the these are typical outputs for the program.

Fig. 39 . Program Analog.

```
..JOB MALLEY ( 10 MINUTES SHOULC DO)
      PROGRAM ANALCG
      CALL EXEC
      END
      SUBROUTINE EXEC
      DIMENSION X(15C),XDOT(100),C(15C),ITITLE(10),JTITLE(10),KTITLE(40)
     1          ,IT(5),IP(10),IG(10),CT(5),XO(100),PR(10),GR(1C),X1(900),
     2           Y1(90C),X2(900),Y2(900),X3(900),Y3(900),X4(9C0),Y4(900),
     3           X5(90C),Y5(900),A(100)
      DO 1 J=1,150
      X(J) = C.
    1 C(J) = 0.
      DO 1C01 J=1,10C
 1001 XO(J) = 0.
      DO 1002 J=1,1C
      ITITLE(J) = 8H
 1002 JTITLE(J) = 8H
      DO 1003 J=1,40
 1003 KTITLE(J) = 8H
      NRC = 0
      PRINT 200
  200 FORMAT(1H1)
  201 FORMAT(/)
  202 FORMAT(//)
  203 FORMAT(///)
      READ 102, (ITITLE(J),J=1,5)
  102 FORMAT(10A8)
      READ 101,N,ITEST
  101 FORMAT(I2,A8)
      ICHECK = 8H EQUATIC
      IF(ICHECK - ITEST) 2,3,2
    2 PRINT 204
  204 FORMAT(18H DATA FORMAT ERRCR)
      STOP 1
    3 READ 101,NR,ITEST
      ICHECK = 8H RUNS
      IF(ICHECK - ITEST)4,6,4
    4 ICHECK = 8H RUN
      IF(ICHECK - ITEST) 7,6,7
    6 IF(NR - 9) 8,8,7
    7 PRINT 204
      STOP 2
    8 READ 101,NT,ITEST
      ICHECK = 8H TITLE C
      IF(ICHECK - ITEST) 10,9,10
    9 IF(NT - 9)11,11,10
   10 PRINT 204
      STOP 3
   11 DO 12 I=1,NT
      READ 102,(JTITLE(J),J=1,10)
   12 PRINT 205,(JTITLE(J),J=1,10)
      IF(NR - 1)10,13,14
   13 PRINT 206
      GO TC 15
   14 PRINT 207,NR
  205 FORMAT(5X, 1CA8)
  206 FORMAT(5X,20H1 RUN IS CALLED FOR.)
  207 FORMAT(5X,I1,21H RUNS ARE CALLED FOR.)
   15 PRINT 203
  300 NRC = NRC + 1
      READ 102,ITEST
      ICHECK1 = 8HZERC CCE
      ICHECK2 = 8HHOLD CCE
      IF(ICHECK1 - ITEST)16,18,16
   16 IF(ICHECK2 - ITEST)17,20,17
   17 PRINT 204
      STOP 4
   18 DO 19 J=1,10C
   19 C(J) = 0.
   20 READ 101,NC,ITEST
```

Fig. 39 . Cont.

```
      ICHECK = 8H COEFF.
      IF(ICHECK - ITEST) 21,22,21
   21 PRINT 204
      STOP 5
   22 IF(NC)23,25,23
   23 DO 24 J=1,NC
      READ 103,(IT(K),CT(K),K=1,5)
  103 FORMAT(5(I4,F12.4))
      DO 24 K=1,5
      ITK = IT(K)
   24 C(ITK) = CT(K)
   25 PRINT 208,NRC
  208 FORMAT(12H RUN NUMBER ,I1,///,46H THE NON-ZERO DATA COEFFICIENTS A
     1RE AS FOLLOWS,/)
      K = 0
      DO 27 J=1,99
      IF(C(J))26,27,26
   26 PRINT 209,J,C(J)
  209 FORMAT(10X2HC(,I2,4H) = ,E12.5)
      K = K + 1
   27 CONTINUE
      IF(K)1027,1026,1027
 1026 PRINT 240
  240 FORMAT(5X4HNONE)
 1027 PRINT 202
      READ 102,ITEST
      ICHECK1 = 8HZERO ICS
      ICHECK2 = 8HFOLD ICS
      ICHECK3 = 8HFOLD FCS
      IF(ICHECK1 - ITEST)30,28,30
   28 DO 29 J=1,N
   29 XO(J) = 0.
      GO TO 35
   30 IF(ICHECK2 - ITEST)31,35,31
   31 IF(ICHECK3 - ITEST) 32,33,32
   32 PRINT 204
      STOP 6
   33 DO 34 J=1,N
   34 XO(J) = X(J)
   35 READ 101,NI,ITEST
      ICHECK = 8H IC CARD
      IF(ICHECK - ITEST)36,37,36
   36 PRINT 204
      STOP 7
   37 IF(NI)38,40,38
   38 DO 39 J=1,NI
      READ 103,(IT(K),CT(K),K=1,5)
      DO 39 K=1,5
      ITK = IT(K)
   39 XO(ITK) = CT(K)
   40 PRINT 210
  210 FORMAT(48H THE NON-ZERO INITITAL CONDITIONS ARE AS FOLLOWS,/)
      K = 0
      DO 42 J=1,N
      IF(XO(J))41,42,41
   41 PRINT 211,J,XO(J)
  211 FORMAT(10X3HXO(,I2,4H) = ,E12.5)
      K = K + 1
   42 CONTINUE
      IF(K)1042,1041,1042
 1041 PRINT 240
 1042 PRINT 202
      READ 102,ITEST
      ICHECK1 = 8HFOLD TIM
      ICHECK2 = 8HREAD TIM
      IF(ICHECK1 - ITEST)43,46,43
   43 IF(ICHECK2 - ITEST)44,45,44
   44 PRINT 204
      STOP 8
   45 READ 104,TO,TF
```

Fig. 39. Cont.

```
 104 FORMAT(10F8.4)
  46 READ102,ITEST
     ICHECK1 = 8HCOMPUTE
     ICHECK2 = 8HHOLD STE
     ICHECK3 = 8HREAD STE
     IF(ICHECK1 - ITEST)48,47,48
  47 INDIC1 = 1
     READ 101,KP,ITEST
     ICHECK = 8H IS ORDE
     IF(ICHECK - ITEST)50,59,50
  48 IF(ICHECK2 - ITEST)49,59,49
  49 IF(ICHECK3 - ITEST)50,51,50
  50 PRINT 204
     STOP 9
  51 INDIC1 = 0
     READ 104,(C(J),J=103,109)
     IF(C(103) - TO )52,53,52
  52 PRINT 204
     STOP 10
  53 IF(C(104))54,52,54
  54 NDT = 1
     IF(C(105) - TF)55,59,55
  55 IF(C(106))56,52,56
  56 NDT= 2
     IF(C(107) -TF)57,59,57
  57 IF(C(108))58,52,58
  58 NDT = 3
     IF(C(109) -TF)52,59,52
  59 PRINT 212,TO,TF
 212 FORMAT(31H THE TIMING DATA ARE AS FOLLOWS ,//
    1        5X,15HINITIAL TIME = ,E11.5,/
    2        5X,15HFINAL TIME    = ,E11.5)
     IF(INDIC1 - 1)61,60,61
  60 PRINT 213,KP
 213 FORMAT(5X,35HTHE STEP SIZE IS COMPUTED, BASED ON ,/
    1        5X,31HSMALLEST VARIABLE OF ORDER 1.0E ,I2)
     AP = 10.0**KP*1.0E-04/(TF - TO)
     DO 1060 J=1,N
1060 A(J) = AP
     A(100) = TF -TO
     C(104) = A(100)*1.0E-05
     GO TO 63
  61 NDTT = NDT*2
     DO 62 K=1,NDTT,2
  62 PRINT 214,C(103+K),C(102+K),C(104+K)
 214 FORMAT(5X,15HSTEP SIZE      = ,E11.5,10H FROM T = ,E11.5,8H TO T = ,
    1         E11.5)
  63 PRINT 202
     READ 102,ITEST
     ICHECK = 8HHOLD PRI
     IF(ICHECK - ITEST)64,74,64
  64 ICHECK = 8HREAD PRI
     IF(ICHECK - ITEST)65,1064,65
1064 READ 101,NP,ITEST
     ICHECK = 8H VARIABL
     IF(ICHECK - ITEST)65,66,65
  65 PRINT 204
     STOP 11
  66 IF(NP)67,74,67
  67 IF(NP - 10)68,68,65
  68 READ 1104 (IP(K),K=1,NP)
1104 FORMAT(10I4)
     READ 101,NT,ITEST
     ICHECK = 8H TITLE C
     IF(ICHECK - ITEST)65,69,65
  69 IF(NT)70,73,70
  70 IF(NT - 4)71,71,65
  71 DO 72 I=1,NT
     K = (I - 1)*10 +1
     KP9 = K + 9
```

Fig. 39. Cont.

```
   72 READ 102,(KTITLE(J),J=K,KP9)
   73 READ 101,INCPR,ITEST
      ICHECK = 8H INCREME
      IF(ICHECK - ITEST)65,74,65
   74 READ 102,ITEST
      ICHECK = 8HHOLD GRA
      IF(ICHECK - ITEST)75,81,75
   75 ICHECK = 8HREAD GRA
      IF(ICHECK - ITEST)76,77,76
   76 PRINT 204
      STOP 12
   77 READ 101,NG,ITEST
      ICHECK = 8H GRAPHS
      IF(ICHECK - ITEST)76,78,76
   78 IF(NG)79,81,79
   79 IF(NG - 5)80,80,76
   80  NG2 = NG*2
      READ 1104,(IG(K),K=1,NG2)
      READ 101,INCGR,ITEST
      ICHECK = 8H INCREME
      IF(ICHECK - ITEST)76,81,76
   81 PRINT 215
  215 FORMAT(17H PRINT SUMMARY---,/)
      IF(NP)83,82,83
   82 PRINT 216
  216 FORMAT(5X,11HNO PRINTOUT)
      GO TO 84
   83 PRINT 217,INCPR
  217 FORMAT(5X,I2,29H INCREMENTS BETWEEN PRINTOUTS,/
     1            5X,    25HTHE VARIABLES PRINTED ARE,/ )
      PRINT 218,(IP(J),J=1,NP)
  218 FORMAT(10X,2HX(,I3,1H))
      DO 1084 J=1,NP
      IF(IP(J))1084,1085,1084
 1084 CONTINUE
      GO TO 84
 1085 PRINT 1217
 1217 FORMAT (/,5X,22H* X(0) REPRESENTS TIME )
   84 PRINT 202
      PRINT 219
  219 FORMAT(17H GRAPH SUMMARY---,/)
      IF(NG)86,85,86
   85 PRINT 220
  220 FORMAT(5X, 9HNO GRAPHS)
      GO TO 2089
   86 PRINT 221, INCGR
  221 FORMAT(5X,I2,26H INCREMENTS BETWEEN POINTS,/ )
      PRINT 241,IG(1),IG(2)
  241 FORMAT (10X,13HGRAPH A IS X(,I3,8H) VS. X(,I3,1H) )
      IF(NG - 2)87,1087,1087
 1087 PRINT 242,IG(3),IG(4)
  242 FORMAT (10X,13HGRAPH B IS X(,I3,8H) VS. X(,I3,1H) )
      IF(NG - 3)87,1088,1088
 1088 PRINT 243,IG(5),IG(6)
  243 FORMAT (10X,13HGRAPH C IS X(,I3,8H) VS. X(,I3,1H) )
      IF(NG - 4)87,1089,1089
 1089 PRINT 244,IG(7),IG(8)
  244 FORMAT (10X,13HGRAPH D IS X(,I3,8H) VS. X(,I3,1H) )
      IF(NG - 5)87,1090,1090
 1090 PRINT 245,IG(9),IG(10)
  245 FORMAT (10X,13HGRAPH E IS X(,I3,8H) VS. X(,I3,1H) )
   87 DO 2087 J=1,NG2
      IF(IG(J))2087,2088,2087
 2087 CONTINUE
      GO TO 2089
 2088 PRINT 1217
 2089 PRINT 200
      PRINT 205,(ITITLE(J),J=1,5)
      PRINT 223,NRC
  223 FORMAT(5X,10HRUN NUMBER,I2,///)
```

```
      IF(NP)88,90,88
   88 IF(NT)89,90,89
   89 PRINT 224,(KTITLE(J),J=1,KP9)
  224 FORMAT(2X,9(A8,4X),A8)
      PRINT 201
   90 T = TO
      DT = C(104)
      DO 91 J=1,N
   91  X(J) = XO(J)
      LINES = 0
      NOPTS = 0
      NUMPTS = 0
  301 IF(NP)302,312,302
  302 IF(NOPTS)303,308,303
  303 IF(XMODF(NOPTS,50*INCPR))304,306,304
  304 IF(XMODF(NOPTS,10*INCPR))305,307,305
  305 IF(XMODF(NOPTS,    INCPR))312,308,312
  306 PRINT 200
  307 PRINT 201
  308 CALL DERIV(T,X,XDOT,C)
      LINES = LINES + 1
      DO 311 J=1,NP
      IF(IP(J))310,309,310
  309 PR(J) = T
      GO TO 311
  310 IPJ = IP(J)
      PR(J) = X(IPJ)
  311 CONTINUE
      PRINT 225,(PR(J),J=1,NP)
  225 FORMAT(10(1X,E11.5))
  312 IF(NG)313,318,313
  313 IF(XMODF(NOPTS,INCGR))318,314,318
  314 IF(XMODF(NOPTS,INCPR))1315,1314,1315
 1315 CALL DERIV(T,X,XDOT,C)
 1314 DO 317 J=1,NG2
      IF(IG(J))316,315,316
  315 GR(J) = T
      GO TO 317
  316 IGJ = IG(J)
      GR(J) = X(IGJ)
  317 CONTINUE
      NUMPTS = NUMPTS + 1
      Y1(NUMPTS) = GR(1)
      X1(NUMPTS) = GR(2)
      Y2(NUMPTS) = GR(3)
      X2(NUMPTS)= GR(4)
      Y3(NUMPTS) = GR(5)
      X3(NUMPTS) = GR(6)
      Y4(NUMPTS) = GR(7)
      X4(NUMPTS) = GR(8)
      Y5(NUMPTS) = GR(9)
      X5(NUMPTS) = GR(10)
  318 NOPTS = NOPTS + 1
      IF(LINES - 250)1319,1318,1318
 1318 PRINT 1216
 1216 FORMAT (//24H STOP AT 250 PRINT LINES)
      GO TO 341
 1319 IF(T - 1.E+04)320,319,319
  319 PRINT 226
  226 FORMAT(//19H STOP AT T = 10,000)
      GO TO 341
  320 IF(NOPTS - 10000 )322,321,321
  321 PRINT 227
  227 FORMAT(//26H STOP AT 10,000 INCREMENTS)
      GO TO 341
  322 IF(NUMPTS - 900)324,323, 323
  323 PRINT 228
  228 FORMAT(//25H STOP AT 900 GRAPH POINTS)
      GO TO 341
  324 IF(T - TF)326,325,325
```

Fig. 39. Cont.

```
 325 PRINT 229
 229 FORMAT(//26H NCRMAL STOP AT FINAL TIME)
     GO TO 341
 326 DO 328 J=1,150
     IF(ABSF(X(J)) - 1.E+04)328,327,327
 327 PRINT 230,J
 230 FORMAT(//11H STCP AT X(,I3,1CH) = 1C,000)
     GO TO 341
 328 CONTINUE
 330 IF(INDIC1)331,332,1331
 331 STOP 13
 332 CALL RKUTTA (N,T,DT,X,C)
     IF(NDT - 1)334,333,334
 333 DT = C(104)
     GO TO 340
 334 IF(NDT - 2)337,335,337
 335 IF(T - C(105))333,336,336
 336 DT = C(106)
     GO TO 340
 337 IF(T - C(105))333,338,338
 338 IF(T - C(107))336,339,339
 339 DT = C(108)
 340 T = T+DT
     GO TC 301
1331 CALL RKUTTA2 (N,T,CT,X,A,C)
     GO TO 301
 341 IF(NG)1341,356,1341
1341 GO TO (342,343,344,345,346,347,348,349,350),NRC
 342 ITITLE(6) = 8H  RUN 1
     GO TO 351
 343 ITITLE(6) = 8H  RUN 2
     GO TO 351
 344 ITITLE(6) = 8H  RUN 3
     GO TC 351
 345 ITITLE(6) = 8H  RUN 4
     GO TO 351
 346 ITITLE(6) = 8H  RUN 5
     GO TO 351
 347 ITITLE(6) = 8H  RUN 6
     GO TO 351
 348 ITITLE(6) = 8H  RUN 7
     GO TO 351
 349 ITITLE(6) = 8H  RUN 8
     GO TO 351
 350 ITITLE(6)  = 8H  RUN 9
 351 LABEL = 4H
     MODCURV = 0
     SFX = 0.
     SFY = 0.
     MINOFFX = 0
     MINOFFY = 0
     LABELNO = 11
     MODE = 0
     ITITLE(7) = 8H GRAPH A
     CALL GRAPH2 (NUMPTS,X1,Y1,8,MODCURV,LABEL,ITITLE,SFX,SFY,
    1            MINOFFX,MINOFFY,LABELNC,MODE)
     IF(NG - 2)356,352,352
 352 ITITLE(7) = 8H GRAPH B
     CALL GRAPH2 (NUMPTS,X2,Y2,8,MODCURV,LABEL,ITITLE,SFX,SFY,
    1            MINOFFX,MINOFFY,LABELNC,MODE)
     IF(NG - 3)356,353,353
 353 ITITLE(7) = 8H GRAPH C
     CALL GRAPH2 (NUMPTS,X3,Y3,8,MCDCURV,LABEL,ITITLE,SFX,SFY,
    1            MINOFFX,MINCFFY,LABELNC,MCDE)
     IF(NG - 4)356,354,354
 354 ITITLE(7) = 8H GRAPH D
     CALL GRAPH2 (NUMPTS,X4,Y4,8,MCDCURV,LABEL,ITITLE,SFX,SFY,
    1            MINOFFX,MINCFFY,LABELNC,MODE)
     IF(NG - 5)356,355,355
 355 ITITLE(7) = 8H GRAPH E
```

Fig. 39. Cont.

```
      CALL GRAPH2 (NUMPTS,X5,Y5,8,MODCURV,LABEL,ITITLE,SFX,SFY,
     1            MINOFFX,MINOFFY,LABELNC,MODE)
  356 PRINT 200
      IF(NRC - NR)300,357,357
  357 RETURN
      END
      SUBROUTINE RKUTTA (N,T,DT,X,C)
      DIMENSION X(150),AK(4,100),XDOT(100),XC(100),CT(4),C(150)
      CT(1) = 0.0
      CT(2) = 0.5
      CT(3) = 0.5
      CT(4) = 1.0
      DO 4 I=1,4
      TC = T + CT(I)*DT
      DO 2 J=1,N
    2 XC(J) = X(J) + CT(I)*AK(I-1,J)
      CALL DERIV (TC,XC,XDOT,C)
      DO 4 J=1,N
    4 AK(I,J) = DT*XDOT(J)
      DO 3 J=1,N
    3 X(J) = X(J) + (AK(1,J)+2.*AK(2,J)+2.*AK(3,J)+AK(4,J))*0.1666666667
      RETURN
      END
      SUBROUTINE RKUTTA2 (N,T,DT,X,A,C)
C             THIS SUBROUTINE IS BASED ON THE METHOD DESCRIBED IN COMM. OF
C             THE A.C.M., JUNE 1960 (PP. 355 -360). IT CHECKS THE
C             TRUNCATION ERROR AT EACH STEP OF THE INTEGRATION AND ADJUSTS
C             THE STEP SIZE ACCORDINGLY. THE ACTUAL RUNGE-KUTTA INTEGRATION
C             IS PERFORMED BY SUBROUTINE RKUTTA, WHICH IS AVAILABLE IN THE
C             USNPGS COMPUTING CENTER. THE ARGUEMENTS ARE,
C                   N = NUMBER OF (FIRST ORDER) EQUATIONS. MAXIMUM N =99.
C                   T = TIME AT START OF INTEGRATION STEP (UPDATED BY THIS
C                       SUBROUTINE AFTER THE COMPLETION OF EACH STEP).
C                  DT = STEP SIZE AT START OF EACH STEP (ALSO UPDATED).
C                X(I) = THE N DEPENDENT VARIABLES (ALSO UPDATED).
C                A(I) = THE SPECIFIED ALLOWABLE ERROR PER UNIT TIME FOR EACH
C                       OF THE DEPENDENT VARIABLES.
C             A(100) IS USED TO ENTER THE TOTAL TIME.
C             NOTE THAT IF NECESSARY DT IS REDUCED FROM THE VALUE STATED
C             IN THE ARGUEMENT UNTIL THE SPECIFIED ACCURACY HAS BEEN
C             ACHIEVED.
      DIMENSION X(150),XS(100),X22(100),X2(100),A(100),C(150)
      TS = T
      H = 2.0*DT
      DO 1 I = 1,N
      X2(I) = X(I)
    1 X22(I) = X(I)
      CALL RKUTTA (N,TS,H,X22,C)
      H = DT
    2 CALL RKUTTA (N,TS,H,X2,C)
      DO 3 I=1,N
    3 XS(I) = X2(I)
      TS = TS + H
      CALL RKUTTA (N,TS,H,XS,C)
      U2 = 0.03
      DO 6 I=1,N
      E21 = (X22(I) -XS(I))*0.06666666667
      E21R = DIMF(ABSF(E21),ABSF((ABSF(XS(I))+ABSF(X22(I))
     1                  -1.99*ABSF(X(I)))*1.0E-09))
C     THIS CONDITION PREVENTS ROUND-OFF ERROR FROM TAKING CONTROL.
      U = E21R / (A(I)*6.0*H)
      IF (U - U2)6,6,5
    5 U2 = U
    6 XS(I) = XS(I) - E21
      IF (U2 - 1.0)11,7,7
    7 IF (H - A(100)*1.0E-09)8,8,9
    8 DT = A(100)*1.0E-09
C     THIS SETS THE MINIMUM STEP SIZE TO 1.0E-09 TIMES THE TOTAL TIME.
      GO TO 16
    9 DO 10 I= 1,N
```

Fig. 39. Cont.

```
      X22(I) = X2(I)
   10 X2(I) = X(I)
      TS = T
      H = 0.5*H
      GO TO 2
C     THIS RECYCLES THE INTEGRATION IF THE TRUNCATION ERROR IS EXCESSIVE.
   11 IF (U2 - 0.031)12,13,13
   12 DT = 2.0*H
      GO TO 14
   13 DT = SQRTF(SQRTF(0.5/U2))*H
   14 IF (DT - A(100)*0.001)16,16,15
   15 DT = A(100)*0.001
C     THIS SETS THE MAXIMUM STEP SIZE TO 1.0E-03 TIMES THE TOTAL TIME.
   16 T = T + 2.0*H
      C(104) = DT
      DO 17 I =1,N
   17 X(I) = XS(I)
C     THIS UPDATES T AND X(I). DT IS UPDATED BY STATEMENT 8,12,13 OR 15.
      RETURN
      END
      FUNCTION RELAY (R,DZONE,V)
      IF(ABSF(R) - DZONE)3,3,4
    3 RELAY = 0.0
      RETURN
    4 RELAY = SIGNF(V,R)
      RETURN
      END
      SUBROUTINE DERIV (T,X,XDOT,C)
      DIMENSION X(150),XDOT(100),C(150)
```

Fig. 40. Subroutine Deriv for Program Analog.

```
COMMENTS
C       SUBROUTINE FCR CPEN - CLCSED CYCLE SYSTEM USING A TIMING CKT WHERE
C       TIME TO SWITCH AND TIME TC TURN OFF IS PROP TC THE COMMAND SIGNAL
C       AND THE RELAY OUTPUT IS A CONSTANT IN THE NONLINEAR REGION
        R = C(1)
        ERROR = R - X(1)
        IF(SW1) 599, 599, 600
   599  IF (ABSF(ERRCR - C(2))) 603, 60C, 6C0
   600  SIGN = ABSF(ERRCR)/ERROR
        SW1 = 1.0
        SW2 = 0.C
        IF (T - TS - TIME) 601, 602, 602
   601  DRIVE = C(6) * SW1
        GO TO 604
   602  DRIVE = -C(6)* SW1
        IF(ABSF(XDOT(1)) - .05) 603, 603, 604
   603  SW1 = 0.0
        SW2 = 1.0
        TIME = X(0)
   604  POWER = C(3) * ERROR * SW2 + DRIVE * SW1
        XDOT(1) = X(2)
        XDOT(2) = POWER * C(4) - X(2) * C(5)
        TS = 0.075 * ABSF(R) + 0.05
        TO = 0.092 * ABSF(R) + 0.105
        X(3) = SW1
        X(4) = SW2
        X(5) = DRIVE
        X(6) = POWER
        X(101) = ERRCR
        X(102) = -XDCT(1)
        RETURN
COMMENTS                         THIS IS BLUE DECK ONE
C       X(1)=SYSTEM CUTPUT, X(2)=SYSTEM RATE, C(1)= INPUT, C(2) =1/2WIDTH
C       OF LINEAR ZONE, C(3)=SYSTEM GAIN, C(4)=PLANT GAIN, C(5)=1/TIME C
C       C(6)=MAGNITUDE CF DRIVING VCLTAGE IN NONLIN REGICN
        END
        END
```

Fig. 41. Data input for Program Analog.

```
 KITTERMAN MALLEY THESIS
02 EQUATIONS
04 RUNS
01 TITLE CARDS
OPEN-CLOSED LOOP SERVC SYSTEM WITH SWITCHING = F(TIME, INPUT)
ZERO COEFFS
06 COEFF. CARCS
   1  0.5
   2  0.25
   3  40.00
   4  1.0
   5  5.0
   6  100.0
ZERO ICS
00 IC CARDS
READ TIME DATA
0.0       1.5
COMPUTE STEP SIZE
-1 IS ORDER OF SMALLEST VARIABLE
READ PRINT DATA
07 VARIABLES PRINTED
   0   1   2   3   4   5   6   7
02 TITLE CARDS
  TIME   SYSTEM   SYSTEM   SWITCH   SWITCH   DRIVE   POWER
         OUTPUT    RATE     ONE      TWO
20 INCREMENTS BETWEEN PRINTS
READ GRAPH DATA
02 GRAPHS
   1   0 102 101
01 INCREMENTS BETWEEN POINTS
HOLD COEFFS
01 COEFF. CARDS
   1  1.0
ZERO ICS
00 IC CARDS
HOLD TIME DATA
HOLD STEP SIZE
HOLD PRINT DATA
HOLD GRAPH DATA
HOLD COEFFS
01 COEFF. CARDS
   1  1.5
ZERO ICS
00 IC CARDS
HOLD TIME DATA
HOLD STEP SIZE
HOLD PRINT DATA
HOLD GRAPH DATA
HOLD COEFFS
01 COEFF. CARDS
   1  2.0
ZERO ICS
00 IC CARDS
HOLD TIME DATA
HOLD STEP SIZE
HOLD PRINT DATA
HOLD GRAPH DATA
```

Fig. 42. Typical output for Program Analog.

```
    OPEN-CLOSED LOOP SERVO SYSTEM WITH SWITCHING = F(TIME, INPUT)
    4 RUNS ARE CALLED FOR.


RUN NUMBER 1

THE NON-ZERO DATA COEFFICIENTS ARE AS FOLLOWS

          C( 1) =    .50000E+00
          C( 2) =    .25000E+00
          C( 3) =    .40000E+02
          C( 4) =    .10000E+01
          C( 5) =    .50000E+01
          C( 6) =    .10000E+03


THE NON-ZERO INITITAL CONDITIONS ARE AS FOLLOWS

    NONE


THE TIMING DATA ARE AS FOLLOWS

    INITIAL TIME =  .00000E+00
    FINAL TIME   =  .15000E+01
    THE STEP SIZE IS COMPUTED, BASED ON
    SMALLEST VARIABLE OF ORDER 1.0E-1


PRINT SUMMARY---

     5 INCREMENTS BETWEEN PRINTOUTS
    THE VARIABLES PRINTED ARE

          X(   0)
          X(   1)
          X(   2)
          X(   3)
          X(   4)
          X(   5)
          X(   6)

    * X(0) REPRESENTS TIME


GRAPH SUMMARY---

    NO GRAPHS
```

Fig. 42. Cont.

KITTERMAN MALLEY THESIS
RUN NUMBER 1

TIME	SYSTEM OUTPUT	SYSTEM RATE	SWITCH ONE	SWITCH TWO	DRIVE
.00000E+00	.00000E+00	.00000E+00	.00000E+00	.10000E+01	-.10000E+03
.93000E-03	-.43178E-04	-.92784E-01	.10000E+01	.00000E+00	-.10000E+03
.12810E-01	-.80324E-02	-.12408E+01	.10000E+01	.00000E+00	-.10000E+03
.27810E-01	-.36938E-01	-.25963E+01	.10000E+01	.00000E+00	-.10000E+03
.42810E-01	-.85432E-01	-.38538E+01	.10000E+01	.00000E+00	-.10000E+03
.57810E-01	-.15210E+00	-.50205E+01	.10000E+01	.00000E+00	-.10000E+03
.72810E-01	-.23563E+00	-.61029E+01	.10000E+01	.00000E+00	-.10000E+03
.87810E-01	-.33479E+00	-.71070E+01	.10000E+01	.00000E+00	-.10000E+03
.10281E+00	-.44847E+00	-.80386E+01	.10000E+01	.00000E+00	-.10000E+03
.11781E+00	-.57562E+00	-.89029E+01	.10000E+01	.00000E+00	-.10000E+03
.13281E+00	-.71525E+00	-.97048E+01	.10000E+01	.00000E+00	-.10000E+03
.14500E+00	-.83727E+00	-.10313E+02	.10000E+01	.00000E+00	-.10000E+03
.14500E+00	-.83730E+00	-.10314E+02	.10000E+01	.00000E+00	-.10000E+03
.14500E+00	-.83730E+00	-.10314E+02	.00000E+00	.10000E+01	-.10000E+03
.14500E+00	-.83731E+00	-.10313E+02	.00000E+00	.10000E+01	-.10000E+03
.14502E+00	-.83755E+00	-.10311E+02	.00000E+00	.10000E+01	-.10000E+03
.14579E+00	-.84538E+00	-.10231E+02	.00000E+00	.10000E+01	-.10000E+03
.15715E+00	-.95485E+00	-.90469E+01	.00000E+00	.10000E+01	-.10000E+03
.17215E+00	-.10790E+01	-.75148E+01	.00000E+00	.10000E+01	-.10000E+03
.18715E+00	-.11805E+01	-.60282E+01	.00000E+00	.10000E+01	-.10000E+03
.20215E+00	-.12601E+01	-.45969E+01	.00000E+00	.10000E+01	-.10000E+03
.21715E+00	-.13187E+01	-.32291E+01	.00000E+00	.10000E+01	-.10000E+03
.23215E+00	-.13574E+01	-.19322E+01	.00000E+00	.10000E+01	-.10000E+03
.24715E+00	-.13771E+01	-.71232E+00	.00000E+00	.10000E+01	-.10000E+03
.26215E+00	-.13791E+01	.42563E+00	.00000E+00	.10000E+01	-.10000E+03
.27715E+00	-.13648E+01	.14777E+01	.00000E+00	.10000E+01	-.10000E+03
.29215E+00	-.13353E+01	.24409E+01	.00000E+00	.10000E+01	-.10000E+03
.30715E+00	-.12920E+01	.33134E+01	.00000E+00	.10000E+01	-.10000E+03
.32215E+00	-.12363E+01	.40941E+01	.00000E+00	.10000E+01	-.10000E+03
.33715E+00	-.11696E+01	.47829E+01	.00000E+00	.10000E+01	-.10000E+03
.35215E+00	-.10933E+01	.53805E+01	.00000E+00	.10000E+01	-.10000E+03
.36715E+00	-.10087E+01	.58384E+01	.00000E+00	.10000E+01	-.10000E+03
.38215E+00	-.91707E+00	.63085E+01	.00000E+00	.10000E+01	-.10000E+03
.39715E+00	-.81983E+00	.66436E+01	.00000E+00	.10000E+01	-.10000E+03
.41215E+00	-.71817E+00	.68969E+01	.00000E+00	.10000E+01	-.10000E+03
.42715E+00	-.61331E+00	.70722E+01	.00000E+00	.10000E+01	-.10000E+03
.44215E+00	-.50638E+00	.71735E+01	.00000E+00	.10000E+01	-.10000E+03
.45715E+00	-.39845E+00	.72053E+01	.00000E+00	.10000E+01	-.10000E+03
.47215E+00	-.29054E+00	.71725E+01	.00000E+00	.10000E+01	-.10000E+03
.48715E+00	-.18358E+00	.70798E+01	.00000E+00	.10000E+01	-.10000E+03
.50215E+00	-.78422E-01	.69325E+01	.00000E+00	.10000E+01	-.10000E+03
.51715E+00	.24149E-01	.67358E+01	.00000E+00	.10000E+01	-.10000E+03
.53215E+00	.12343E+00	.64949E+01	.00000E+00	.10000E+01	-.10000E+03
.54715E+00	.21880E+00	.62152E+01	.00000E+00	.10000E+01	-.10000E+03
.56215E+00	.30972E+00	.59018E+01	.00000E+00	.10000E+01	-.10000E+03
.57715E+00	.39571E+00	.55599E+01	.00000E+00	.10000E+01	-.10000E+03
.59215E+00	.47640E+00	.51946E+01	.00000E+00	.10000E+01	-.10000E+03
.60715E+00	.55146E+00	.48107E+01	.00000E+00	.10000E+01	.10000E+03
.62215E+00	.62065E+00	.44128E+01	.00000E+00	.10000E+01	.10000E+03
.63715E+00	.68380E+00	.40054E+01	.00000E+00	.10000E+01	.10000E+03